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A SMALL WORLD PERSPECTIVE ON URBAN SYSTEMS

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ABSTRACT

The theory of small world network as initiated by Watts and Strogatz 1998 has drawn new insights on spatial analysis as well as to systems theory. Its concepts and methods are particularly relevant to geography where spatial interaction is mainstream, and where interactions can be described and studied using large volume of exchanges or similarity matrices.

Networks indeed organize through direct links or by indirect paths, inducing topological proximities simultaneously involving spatial, social, cultural or organizational dimensions. Network synergies build over similarities and are fed by complementarities between or inside cities, the two effects potentially amplifying each other according to the "preferential attachment" hypothesis that has been explored in a number of different scientific fields (Barabàsi, Albert, 1999; Barabàsi, 2002; Newmann, Barabàsi, Watts, 2006). In fact, according to Barabási and Albert (1999), the high level of hierarchy observed in "scale-free networks" results from "preferential attachment" which characterizes the development of networks: new connections appear preferentially close nodes already having the largest number of connections because in this way the improvement in the network accessibility of the new connection will probably be greater. But at the same time, network regions gathering dense and numerous weak links (Granovetter, 1973, 1985), or network entities acting as bridges between several components (Burt, 2004) offer a higher capacity for urban communities to benefit from opportunities and create future synergies. Several methodologies have been suggested on how such denser and more coherent regions (also called communities or clusters) in term of links can be identified (Watts & Strogatz, 1998; Watts, 1999; Barabasi & Albert, 2000; Barabasi, 2003; Auber *et al*. 2003, Newmann *et al*., 2006). These communities not only possess a high level of dependency between their member entities, but also show a low level of "vulnerability" allowing for numerous redundancies (Burt, 2000, 2005).

The SPANGEO project 2005-2008 (SPAtial Networks in GEOgraphy), gathering a team of geographers and computer scientists, has conducted empirical studies to survey concepts and measures developed in other related fields such as physics, sociology or communication science. The relevancy and potential interpretation of weighted or non weighted measures on edges and nodes were examined and analyzed at different scales (intra-urban, inter-urban or both). New classification and clustering schemes based on the relative local density of subgraphs were developed. The article describes how these notions and methods bring a contribution on a conceptual level, in terms of measures, delineations, explanatory analysis and visualization of geographical phenomena.

KEYWORDS

Urban Systems, networks, graphs, Small Worlds, multi-level approach, multi geographical scale.

INTRODUCTION

The increasing complexity of spatial organizations, in particular of cities, challenges the study of spatial distribution. Places tend to be seen more and more as nodes in specialized networks and their future is undoubtedly strongly dependant to their position in these networks (Castells, 1996). Geographical distances and accessibility always matter in spatial dynamics, but in parallel, other distances appear as relevant: social, economical, cultural, organizational distances can also interfere in spatial dynamics. As a consequence, Geography as a science of society in space, must work as defining and measuring how spatial accessibility dynamics influence other kinds of distances, and how processes emerge from spatial or territorial dimensions. From this perspective, in a social science division of work, geography could be defined as the science dedicated to identification of spatial patterns and their implications at specific geographical levels of scale.

These geographical levels are themselves evolving according to systems of social processes. For example, globalization processes are developing as far as transport and communication speed is increasing (and as costs are decreasing), but also according to organizational transformations of society, deriving from actors themselves, like companies and subsidiaries, social groups and institutional arrangements. These types of interactions and their dynamics, explain why globalization has been developed at a worldwide scale for stock exchange, while “global value chains” of production are developing, most of the time, at a continental scale leveraging from “regional” international trade arrangements (Doz et al, 2001; Dicken et al., 2001).

The hypothesis we just laid down suggests that all transformations strongly depend on internal and external relations of components, calling for a study of the relative positions of places in networks and at different geographical scales. Geographical interactions can be studied through multi-dimension networks, which can be considered as graphs. Until now, most geographical network studies were based either on Gravity models (Wilson, 1967) or on general indexes of graph connectivity or accessibility (Kansky, 1963). Graph theory and more particularly Watts’ theory of small word networks (Watts, 1999), can feed this trend of research and enrich spatial analysis as a complement to systems theory. Networks indeed organize through direct links or by indirect paths, inducing topological proximities simultaneously involving spatial, social, cultural or organizational dimensions. Network synergies, either based on the similarities or on the complementarities between or inside cities, create specialized and more or less stable proximities between these entities (Powell, 1990). Interrelations between geographical scales of organization also allow understanding equilibrium or disequilibrium of territories emerging at different geographical scales. The concepts and methods of the small world theory are particularly relevant to geography where spatial interaction is mainstream, and where interactions can be described and studied using large volume of exchanges or similarities matrices. As far as we know, this type of empirical approach combining a conceptual approach of "small world theory" and dedicated tools has not been developed in Geography.

Incidentally, the multi-level analysis and the visualization of graphs bring new questions. How do networks link similar actors reinforcing their social proximities as well as spatial specializations and segregations? In this sense, how do networks (not only transport and communication networks but also social and economic networks) increase anisotropy of space? How far are networks from random processes, creating densely connected groups of nodes separated from others like "Small world" networks (Watts, Strogatz. 1998; Watts, 1999)? What are the properties of these networks and what consequences can we infer on geographical places especially for cities?

In this presentation, we'll stress the advantages to analyze and visualize networks through their network properties (1). Then, we underline topological positions in a static state as well as from a dynamic perspective (2) and transfer them onto properties of cities systems (3). The final purpose discusses levels of organizations, taking into account empirical findings on hierarchical clustering (4).

1- SMALL WORLD THEORY AND SCALE-FREE NETWORKS

This chapter is devoted to the introduction of small world theory and scale-free networks. Laying down these formal notions in a separate section will allow us to freely build the discussion around examples studied by members of the SPANGEO project without having to pause for mathematical digressions.

Graph theory, developed especially by Erdös and his school since the 1950s was based on the concept of "random graph", making the strong assumption that all relationships are likely to occur with equal probability (Erdös, Renyi, 1959, 1960). Put simply a random graph is one where edges are drawn at random playing heads or tails – that is there are no region in the graph where edges have a higher probability to appear, leading to uniformly distributed connections among nodes.

Since the late 1990s, many contributions have proposed alternatives to this model, showing the importance of "small worlds" in the organization of societies and their dynamics, where networks are not seen as homogeneous (Newman, 2000; Newmann Watts, Barabasi, 2006). That is, small world are built from groups of tightly connected *communities*, themselves connected through privileged nodes. The small world phenomena experimentally identified by Milgram (1967) in social networks was observed in many other fields, giving this notion a taste of universality. In biology, for instance, networks of interacting proteins show a small world character, where communities of proteins are embody cell functions (Vespignani, 2003). The internet is another well known and extensively studied example (Adamic, 1999). Two main properties typify these networks:

- The average path length from one individual to another is small and compares to the average path length of random graphs (with the same number of nodes and edges);

- They have a strong propensity to create sub-groups (also called communities or clusters). This is the strongest characteristics of small world networks, which is the result of several micro processes like transitivity, homophily, and range of ties.

Different statistics can be used to assess of the presence of communities in a graph and confirm its small-worldness. The well known *clustering coefficient* introduced by Watts relates to transitivity and roughly measures the probability that neighbors of a node are connected. The work by Watts and Strogatz has given birth to several extensions and variations of this measure. Ancient work also provides ingredients to investigate networks. The Jaccard measure (Jaccard 1901), for one, assigns a value to an edge depending on the number of common neighbors its incident nodes has; the topological overlap matrix index (Ravazs et al. 2002) is similar to the Jaccard index and was introduced in the context of bioinformatics. As mentioned earlier, small worlds organize into subgroups and are connected through privileged nodes acting as *bridges*; (Burt 2000, 2005) refers to this phenomenon as *structural holes*. Many authors have designed metrics aiming at the identification of these bridges, the most used certainly being the *betweeness centrality* defined by Freeman (1977) (see also Brandes, 2001). The *strength metric* defined in Auber et al. (2003) also aims at the identification of bridges. (See Sallaberry & Melançon (2008) for a short survey on edge metrics.)

In parallel to Watts and Strogatz's theory of small worlds, some authors observed and investigated another property of networks, referring to them as being scale-free. These networks organize into an order hierarchy with respect to the degree of nodes and evolve along a basic process. According to Barabási and Albert (1999) (see also Barabasi, 2002) who popularized these "scale free networks", this order hierarchy is due to the *"preferential attachment"* that characterizes the development of networks: new links preferentially link to nodes already having the largest number of links, favoring the hierarchy. In reality, we can find earlier such explanations of the Zipf law proposed by Simon (1955), referring to the *"Yule process"* (Yule, 1925). The process thus induces the degree distribution of nodes to follow a power-law, that is the frequency of nodes of degree k is approximately given by $p(k) \sim c \exp(-\gamma k)$. In other words, a sharp inequality exists between strongly central entities and others who cling to these centralities. (The interested reader is referred to (Bornholdt and Schuster 2003).)

The scale-freeness of spatial networks makes sense, as the forthcoming sections shall explain. However, it makes things much harder for their visual exploration and (automatic) analysis. Most often, networks show both character simultaneously. That is, networks do contain small communities of interacting entities, while the overall organization of the network is dominated by a scale-free process. The high degree nodes thus, make it hard to identify communities or bridges and hide the small world structure. Hybrid approaches have to be designed in order to reveal the structure and properties of these complex networks.

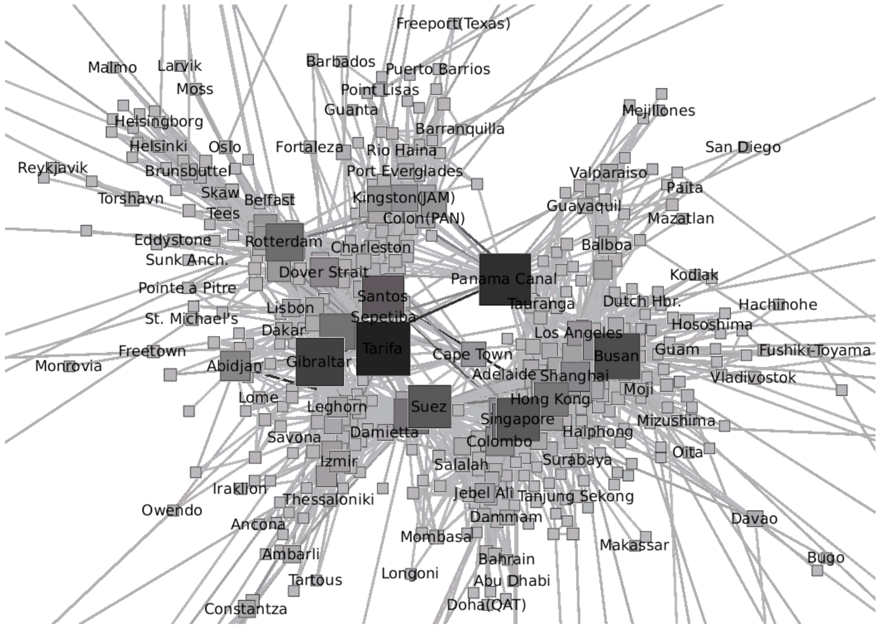
2- VISUALIZING TOPOLOGICAL PROXIMITIES IN GEOGRAPHICAL NETWORKS

Visualizations are common tools for geographers, using cartography to stress geographical organizations and patterns. Nevertheless, as far as geographical distance is not the only one factor constraining the system and its dynamics, a topological representation can be much more useful to highlight organizational structures.

A basic geographical example, is the worldwide containers traffic networks, where the visualization allows for a good overview of the overall structure (Fig. 1) (Ducruet, Rozenblat, Zaidi, 2010). On the one hand, the representation ignores the geographical coordinates of ports. On the other hand, their location in the graph is determined by applying a layout based on topological proximities for maritime exchanges. Therefore, the geographical situation is preserved due to the continuum of movements from one port to another.

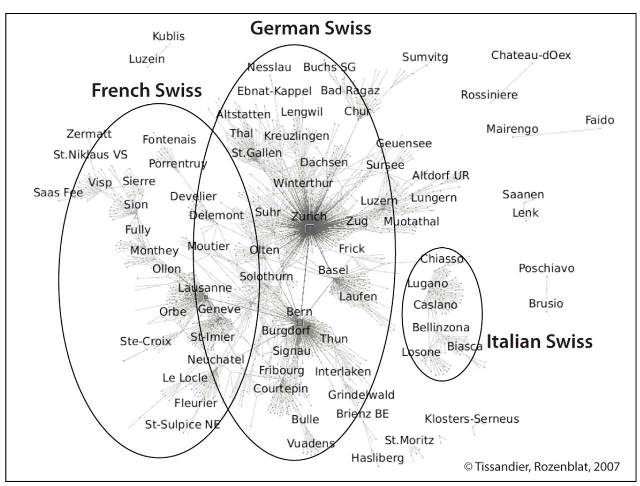
Fundamentally, two main sub-systems exist: Asia-Pacific, and Europe-Atlantic. These sub-systems are connected through a limited number of nodes: Panama and Suez canals. Removing those two global pivots would thus split the world system in two parts, although one may notice some other links (e.g. Magellan Strait, and Cape of Good Hope) but those remain very limited compared to the main trunk lines concentrating at the two canals. Also, Port ranges appear as the Scandinavian range in the top left of the figure 1, close to American ranges.

Figure 1: Worldwide container traffic Network 2006



Another example maintaining some geographical position into the topologic network representation is the topic of commuters (Fig.2). The example of Switzerland shows especially the linguistic proximities between municipalities with 3 visible isolated zones: French Swiss speaking at left, Italian speaking at right, and Swiss German in the center, cut into two parts between a system around Bern (the political federal capital), and another around Zürich (the economic capital).

Figure 2: Commuters between Swiss Municipalities in 2000



3- NETWORKS' PROPERTIES AS CITIES SYSTEMS' PROPERTIES

Urban systems as small worlds:

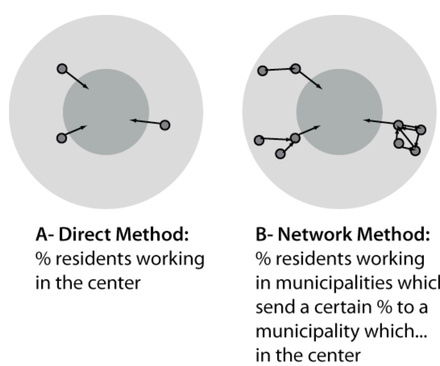
The small world property has a clear interpretation for urban systems in general. The short average distance between entities can be seen as a consequence of redundant paths through different routes between cities' networks, strengthening direct and/or indirect interaction. This mutual interaction is mainly through "clusters" which can form inside cities or towns, heavily connected by groups.

Empirical evidences of these networks are founded on some basic properties reinforcing these strengths (Monge, Contractor, 2003) which, in transposition, could make sense for geographical networks organizations:

- Transitivity: This elementary property was early shown by sociologists (Weimann, 1980): If an individual A is linked to B, and B to C, then the likelihood is high that A should be linked to C.

  In geographical perspectives, we can transpose this property to interaction and interdependencies effects. If a place A is closely related and interdependent to B, and B to C then A is interdependent to C. For example, commuter exchanges between municipalities create this kind of chain dependencies (Fig.3). If a factory employing several thousands people in A suddenly closes, it will affect residential population of B sending commuters in A, but in consequence, it also will affect C sending commuters to B. Sometimes, this transitive relation is materialized by real exchanges (like commuters), but even if there's not such direct materialized exchanges, interdependencies occur as well through indirect effects. So some local chains create tangled networks allowing to very integrated local systems of territories (Rozenblat et al., 2009).

Figure 3: Urban Areas delineation using transitivity property



At the inter-urban scale, interdependencies make also sense in mutual exchanges (symmetric or not), increasing proximities in terms of space (local or regional economical networks) or in terms of specialization (worldwide clusters linked through multinational firms' global value chains or through research centers). These new proximities stimulate growth of exchanges, strengthening closeness of some specific groups of cities or territories in spatial, economic, social or cultural proximities.

- Homophily: Networks between actors, territories or cities with similar attributes have larger probability of occurring. These exogenous criteria interact actually with networks because exchanges could also transform places as they transform individuals with imitation behaviors and diffusions of many types of ideas, concepts and technologies.
- Range: in parallel to homophily, diversity of elements of places and cities linked by networks seems essential to the reproduction and renewal of territorial systems. This diversity could be seen in terms of exogenous characteristics (for example diversity of activity sectors) as well as endogenous: the diversity of links to other places.

Urban systems as scale-free networks - Hierarchies of individuals

There are evidences in urban systems that innovations spread preferentially from the largest cities, already concentrating multiple networks, to the smallest ones (Batty, 2005; Pumain, 2006). At a micro level of individual behavior, these processes are in a large part due to the search of security rather than maximization, in a context of partial information in game theory (Luce, Raiffa, 1957). The "bounded rationality" mainly developed by Simon (1957, 1972, 1999) explains why, in a context of random probability of meeting opportunities, individuals choose the first satisfactory opportunity, rather than the best one. These trends lead to a strengthening of major cities, where opportunities are more diversified and where interactions are maximized, decreasing transaction costs (Coase, 1937). This process allows some specific properties of centralities, equivalence or structural equivalence positions for places or for groups of places according to positions of located individuals.

- Centralities: inequalities between places in term of relative position in networks, are obviously dependent on accessibility or reachability into some set of places, in a certain context and at a certain period (Bretagnolle, 1999). Many kinds of indexes bring close views on these central positions, like *Degree* (number of links), *Reachability* (distance to all other points), *Betweeness Centrality* (number of shortest paths passing by a point), or *Generalized Strahler* index (Delest, Aubert, 2003; Auber et al., 2004) measuring how many trees could be built from a node : every measure can be weighted or not (Wasserman, Faust, 1994;

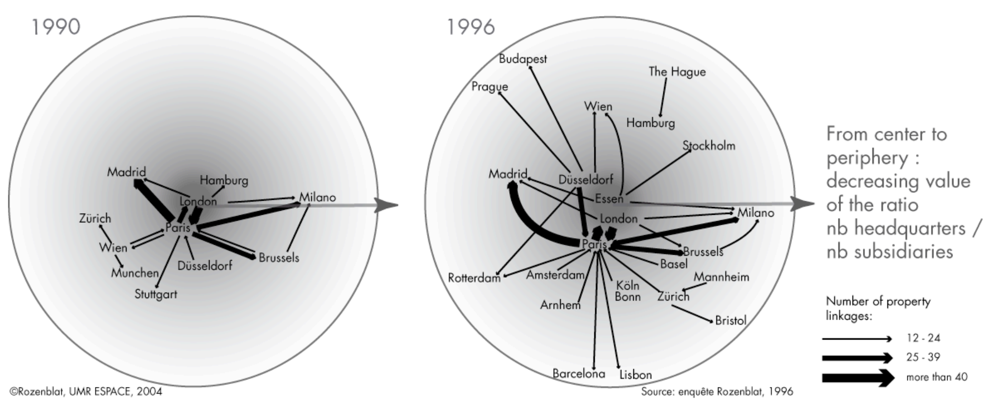
Newmann, 2003; Brandes, Erlebach, 2005). They are all very useful to compare node positions in a graph, or even position of the same node in different graphs (multiplex graph).

- Dependencies and power: when social, economic or networks are oriented, either reflecting ownership between enterprises, decision-making, or hierarchy. The balance between in-degree ($Deg^{in}{}_i$) and out Degree ($Deg^{out}{}_i$) can be a measure of such dependencies or power. Thus, an index of power P_i can be calculated for each city (Rozenblat, 2004):

$$P_i = Deg^{out}{}_i - Deg^{in}{}_i$$

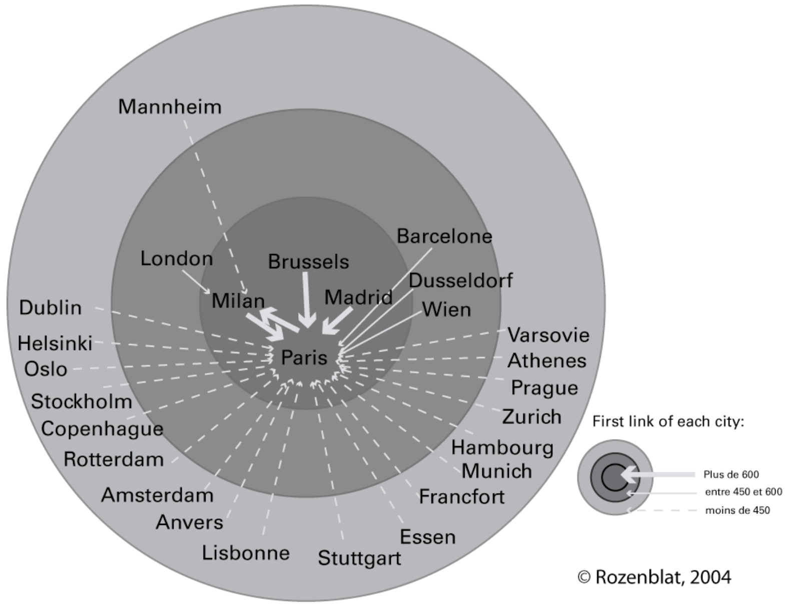
For example, this methodology can easily underline cities of power in multinational firms networks according to the opposition between headquarters and subsidiaries (Fig.4). The ties are oriented and the centrality in the graph is calculated by the share of the number of Headquarters (representing the out Degree) by the number of subsidiaries (in Degree). Between 1990 and 1996, the graph has spread all over Europe in particular areas like south and Eastern Europe. London, with a high share of headquarter/subsidiaries, is the most central with many American Headquarters controlling subsidiaries in the continent. But Paris seems much more connected to other European big cities.

Figure 4: Hierarchy of European cities according to their control of Multinational firms 1990-1996



Also, one can highlight dual positions where some cities are only linked to one other. According to Nystuen and Dacey's methodology (1961), one can build some conceptual hierarchy with the highest link of each city going to a bigger city. All these links create some "regional systems". So, hierarchy can also emerge from non-oriented ties according to weighted dependencies. Fig. 5 shows this construction applied to European cities connections through complete graphs built inside each firm of a sample of 100 groups in 1996 (Fig.5). The first link of the majority of European cities is then oriented to Paris, the most connected city according to this kind of firms' graphs.

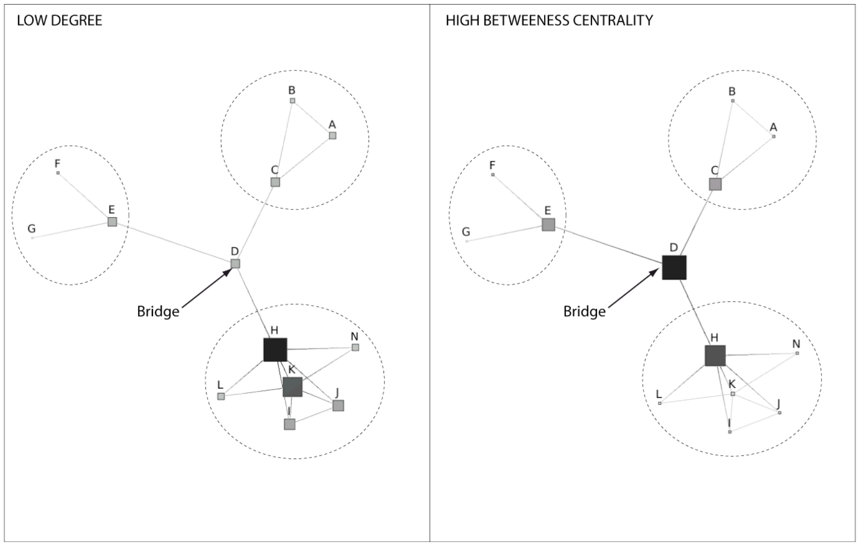
Figure 5: Hierarchy of European cities according to their positions in Multinational firms in 1996



- Equivalence and Structural equivalence: Organizational structure is viewed as a pattern of relations among positions. White, Boorman and Breiger (1976) and Burt (1982) have developed positional theories of *structural equivalence* According to Monge and Contractor, this theory "*argues that people maintain attitudes, values, and beliefs consistent with their organizational positions irrespective of the amount of communication that they have with others in their organizational networks*" (2003, p.19). In a geographical perspective, we could argue that places like cities, are formed by tangled and overlapped social, economic and institutional networks which put them in some equivalent positions as well. For example, cities of the same country are equivalent faced to their political national capital in many ways from hierarchical administration to hierarchical economy and culture. On the other hand, national capitals are in such structural equivalence faced to their respective national urban systems even if some national networks are more hierarchical than others.

  The confrontation of weighted degree and betweeness centrality of each city in every level allows underline *Bridge cities* (Barrat et al., 2005; Guimera et al. 2005). In fact, degree and betweenness centrality are very correlated even if they have different natures: local (for degree) and global (for BC). But a low degree with a high betweeness centrality shows especially the role of bridges between clusters (fig.4-A). We can discuss the threshold to choose in order to select this kind of bridges (fig.4-B). On the other hand, we can enlarge this approach testing all kinds of confrontations of local and global measures.

Figure 6: Identification of bridges



Individuals or organizations acting as bridges (or connectors) can override the "structural holes" strengthening the entire "social capital", but in the first place their own capital (Burt, 2000, 2005). These bridges have a very strong strategic centrality even if they have a small number of links (Guimera et al, 2005). A city can form a "bridge" between several cities when it is located at the interface between different groups or clusters, or between different units (as a relay between its national cities and abroad, between continental and the world).

In dynamic terms, such urban placements require high flexibility in networks and strong adaptability of cities and reactivity for their actors. In contrast, too strong a hierarchy between nodes freezes the system, but leaves individuals excluded from any network. The limitations of networks between what is integrated and not integrated, are highly constitutive of their operation and evolution (Barabasi, 2003).

Hierarchies of groups

If we assume that interaction implies interdependencies, then groups of cities which exchange more together than with others, are more dependent with each other. They constitute groups that can be defined in different ways (we'll see below). But whatever method of classification is used, these groups imply some relative closure, due in part from transitivity property saw previously. For other cases, they constitute some star around a central node, linked itself to outside (a Hub). So, a kind of "cohesion" measuring the relative density of the group's graph can be interpreted as strength of interdependencies. As in classical classifications, some nodes (like cities) contribute more than other ones to the cohesion of their groups: this is what we'll call the *"contribution"* of each point to its group. Besides, each city can also have links going out to the group. Then, we can also define a second measure, specifying from the point of view of each node, its participation to several groups: this is what we'll call "*participation"* (Guimera et al., 2005). Contribution and participation indexes can be measured as individual shares for one node to one group (or the opposite).

For contribution of a node i to a group g:

$$Contr_i^g = \frac{Deg_i^g}{\sum_{i=1}^{n} Deg_i^g}$$

Where Deg_i^g is the degree of the node i (number of links) in a group g.

For the participation of the node i to a group g:

$$Part_i^g = \frac{Deg_i^g}{Deg_i}$$

Where Deg_i^g is the degree of the node i inside the group g and Deg_i is the total degree of i.

These kinds of indexes can identify hubs in networks. Their role is often concentrated in or between particular communities. Extending Guimera et al. 2005 who introduced z-score and the "*participation coefficient*" of nodes, we take edge weights into account. The z-score of a node measures how much of the node's connection are devoted to its own cluster, while its participating coefficient measures how much its connections cover all other clusters. Because we cluster the graph into a hierarchy of subgraphs, we can moreover measure Guimera et al. indices at every level and study how the indices vary as we go down the hierarchy of clusters. Compared to our indexes the Guimera et al. indices are:

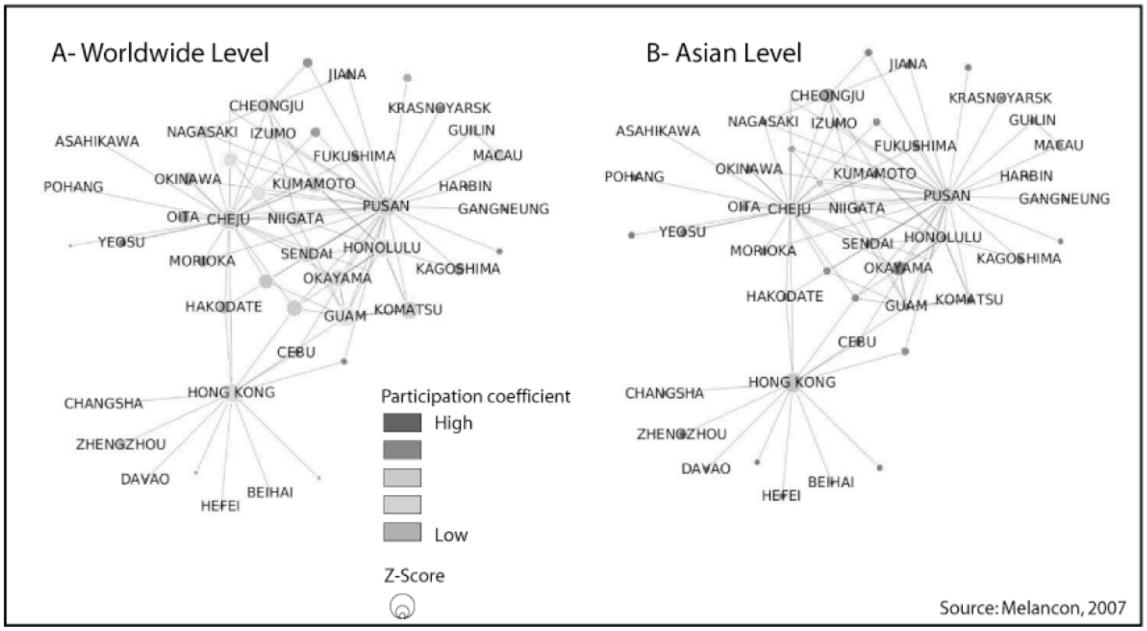
$$z - score_i = Part_i^g$$

where g is the main group of i.

$$Guimera - Participation_i = 1 - \sum_{g=1}^{p} Part_i^{g^2}$$

The figure below provides a good illustration (fig.7). Nodes have been sized according to their z-score (internal connections) and colored according to their participation coefficient (external connections). For example, Hong-Kong has got a higher role at the worldwide level than at the Asian level.

Figure 7: Identification of Hubs by Contribution indexes



For a more global view of each node role, an entropy index H could take into account the balance between contributions of a node to different groups. It measures the share of the maximal entropy: LOG(n). For Entropy of n nodes i contribution to the group g:

$$HContr_g = \sum_{i=1}^{n} \frac{Deg_i^g}{\sum_{i=1}^{n} Deg_i^g} \times LOG\left(\frac{Deg_i^g}{\sum_{i=1}^{n} Deg_i^g} \right) \Big/ LOG(n)$$

Symmetrically, an entropy index H could take into account the balance between participations of different nodes to a group . It measures the share of the maximal entropy: LOG(p). For entropy of the node i participation to different p groups g:

$$HPart_i = \sum_{g=1}^{p} \frac{Deg_i^g}{Deg_i} \times LOG\left(\frac{Deg_i^g}{Deg_i} \right) \Big/ LOG(p)$$

The signification of these entropy indexes could be interpreted as usually for entropy: the degree of information contained into each group or information for each node (Shannon, 1948; Lin, 1999). The higher the information included inside a node or a group, the higher is the resilience and the capability to renew. So, the maximum entropy corresponds to a great variety of links for each node (balanced participation to many groups) or at the level of clusters, various contributions of nodes forming each group (and not only one in the case of star patterns). This variety contains a high disorder and could be interpreted as the systems' high capability to maintain and renew (Bailey, 1990; Burt, 2005). It illustrates for a particular node, the "*strength of Weak Ties*" of Granovetter (1973) where a node has got more opportunity of a large number of weak links relying some differentiated clusters formed by larger links. In those clusters, vicinities' economic links can develop processes of agglomeration economies or network economies. However weak links between clusters are the diversity, which also produces savings network, which allows cities both to operate and renew themselves.

This is the coupling between strong and weak links that allows the reproduction of the urban system and its transformations (Guimera et al., 2005; Uzzi, 1997). We could identify such types of clusters between cities from different kinds of networks (transport, communication, migrations, multinational companies etc.), and we could specify for each city the intensity of its membership in a cluster, assess to what extent these clusters correspond to territorial logic (of continents, country, urban areas), or the logic of economic specialization (cities specialized in the same fields).

Besides diversity, specialization and "closure" are common patterns in networks, according to the general form of "scale free" laws (see before). Then the β index of the "scale free function" can also constitute a norm to measure high concentration (Barabasi, Albert, 1996; Batty, 2005; Paulus et al., 2006). Then the measures of scale free of contributions inside each group could be adopted, but the difficulty remains in defining these groups and their relative positions.

4- MULTI-LEVEL CLUSTERING APPROACHES

Clusters approaches are easy if clusters are disjoint. But clusters can also be formed at different levels, which can be nested, tangled or overlapped. Therefore, a multi-level approach is useful to catch all the dimensions of the processes, to compare them and to identify their complementary roles in the system dynamics.

The "small worlds" networks approach identifies those parts of networks — such as "clusters" — which are the densest and most coherent (Watts & Strogatz, 1998; Watts, 1999; Barabasi & Albert, 2000; Barabasi, 2003; Newmann & al., 2006). Such parts not only demonstrate a high level of dependency between their individual components, but also a low level of "vulnerability" in networks that allow for numerous redundancies (Burt, 2000, 2005). Thus, both through direct links and by more indirect paths, networks organize new topological "proximities" constituted simultaneously of spatial, social, cultural or organizational proximities. In 2004, Newman and Girvan posited a measure of clustering on non-value graphs predicated on the maximization of intra-group connections (Q value). We adapted it to weighted graphs.

G a weighted graph

C= {C1, C2, ..., Cp} a partition of this graph into clusters C1, C2,..., Cp

w(e) is the weight of the edge e (**Passenger Traffic in the case of the air worldwide passenger network, for example**).

The weighted quality measure Q_{val} is defined as follows:

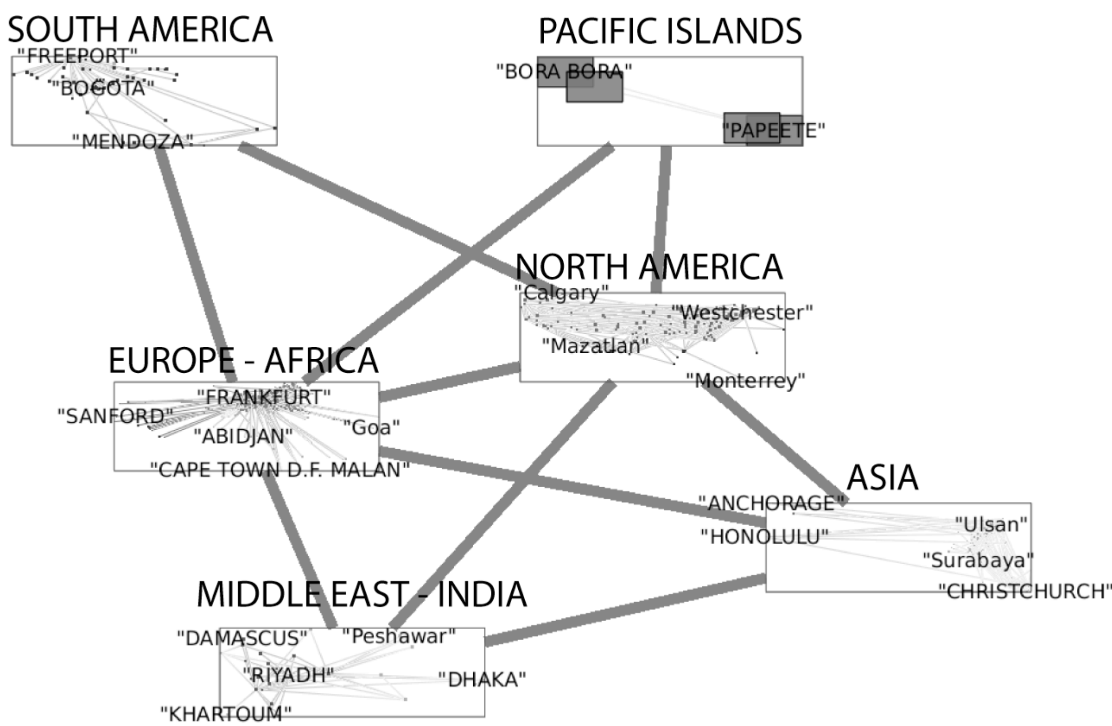
$Q_{val} = \sum i\ Q_{val}(Ci),\ 1 \leq i \leq p.$

and

$Q_{val}(Ci) = (\sum_{e \text{ in } \mathbf{E(Ci)}} \mathbf{w(e)} / \sum_{e \text{ in } \mathbf{E}} \mathbf{w(e)}) - (\sum_{e \text{ in } \mathbf{E(Ci,*)}} \mathbf{w(e)} / \sum_{e \text{ in } \mathbf{E}} \mathbf{w(e)})^2$

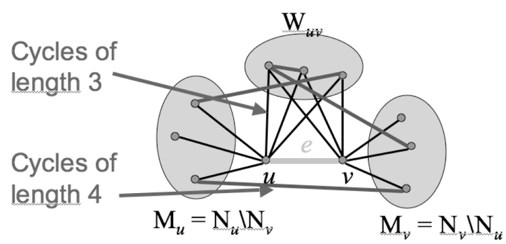
The application of this method to the international air-traffic network — whether it is non-valued (Guimera & al., 2005), or valued, nevertheless demonstrates that groups of cities organize themselves geographically by continents (Fig.8).

Figure 8: Classification of Worldwide air traffic according to Weighted Quality Mesure (Qval)



The clusters are seen as communities and are captured with the use of edge metrics conveying topological properties of the underlying communities. The metrics we use are from Auber et al. (2003) adapted to weighted edges – traffic flows (see also Amiel, Melançon, Rozenblat, 2005). A Strength index is calculated for each edge (corresponding to the density of ties around this edge) (Fig.9). Then Filtering out edges with a low value induces a graph whose connected components form “communities”. This procedure is iterated into each community to define sub-communities and so on at various levels.

Figure 9: Measure of an edge strength index



For this operation, one calculates a ratio based on the number of cycles of length 3 involving an edge e:

$$\gamma_3(e) = \frac{|W_{uv}|}{|M_u| + |M_u| + |W_{uv}|}$$

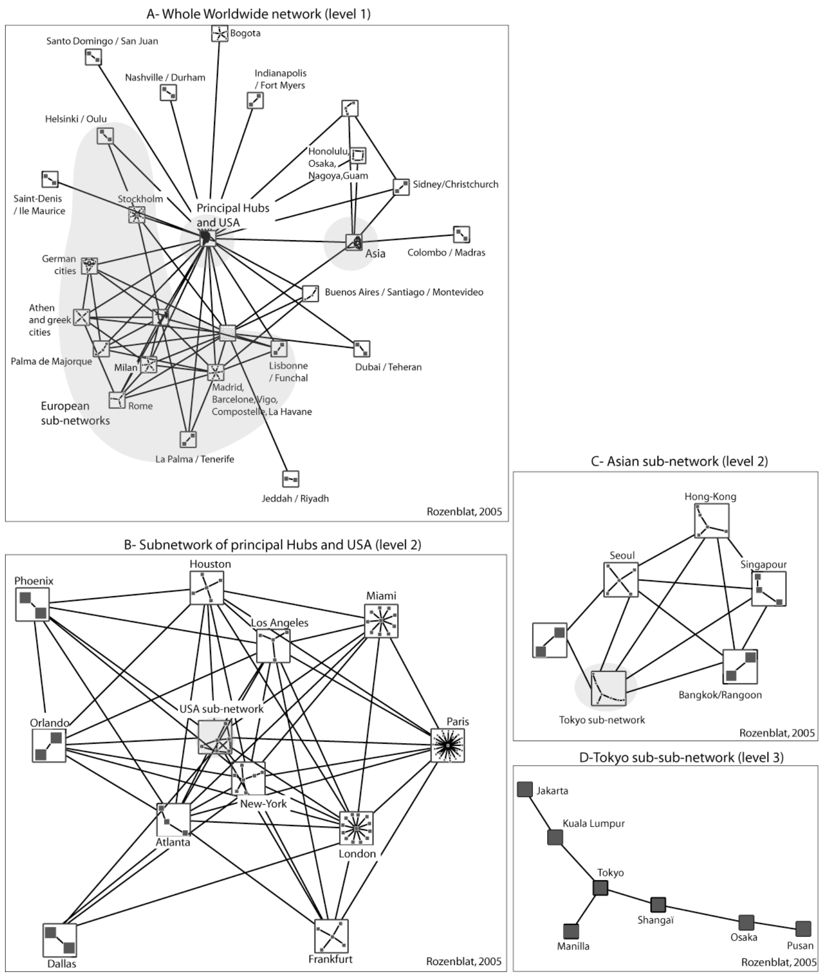
A similar ratio can be defined for cycles of length 4:

$$\gamma_4(e) = s(M_u, W_{uv}) + s(M_v, W_{uv}) + s(M_u, M_v) + s(W_{uv}, W_{uv})$$

Then the strength index is the sum of both: $\Sigma(e) = \gamma_3(e) + \gamma_4(e)$

To ignore edges with low strength indexes, we used a Quality index (Delest et al., 2006). This metric completes Burt’s network constraint, measuring how much an actor is constrained by its direct neighborhood (Burt, 2000, 2005). In both cases, it is expected that edges with a low value act as bridges between tighter communities.

Figure 10: Classification of Worldwide air traffic according to Strength index



As can be expected, communities become more and more cohesive as levels go down. (fig.10). In this approach, the principal hubs are all organized in the same class (fig.10.b). While geographical proximities persist, they are also involved in local hierarchies, either around the national capitals of Europe (Fan, 2006) or they are established by airline companies, as it is the case in Asia (fig.10.c and 10.d).

Because it allows as much weight to the density of the graphs as to the strength of connections, this method appears more able to represent a collection of weak connections forming multi-level groups of cities organized according to their proximities both geographical and functional. The levels are distinguished by the degree of cohesion that they contribute to the network. They may be constituted with respect to geographical imperatives (continents), or to organizational (hubs) or economic imperatives (airline companies). A further improvement to this approach to clustering would presently consist in defining not strictly circumscribed groups but rather a “fuzzy” belonging of each city to various groups, which would allow for the interactions of a given city to be deployed over a wide variety of levels at once. Results could be able to measure how network levels are evolving in a multi-dimensional space comprising the companies' alliances and competitions, the networks economies given by the hub system, the global laws of deregulation, restrictions for environment and the transportation demand.

CONCLUSION

The developing organization of cities into multi-level networks seems to occur not only in the field of transport systems but in organizational systems as well (Dicken & al., 2001). While it is undoubtedly true that agent networks transcend the limitations of geographical scales (Castells, 1996), yet it is also the case — as Whitley has demonstrated (1998) — that the organizational levels of the “global commodity chain” are in the process of crystallizing from local to worldwide scales. In the context of this realization, multinational corporations do not develop within territories that are at once homogenous and open as argued by Ohmae (1990); rather they transversally weave a system of territories — overlapping or not as the case may be — in which everyone produces their own rules and regulations. The structure of the *“transnational field”* continues to be strongly influenced by the *“international field”* and so continues to depend on *“the exchange rates, labor and fiscal regulation, those ‘externalities’ of which the company can take advantage, and the size and quality of the market.”* (Dollfus, 2001, p. 104-105). Considered from this perspective, the continental level is seen to increase its cohesive role by virtue of the creation of free-trade zones, which tends to reinforce continental systems (Ohmae, 1995; Yeung, 2002; Rugman, 2001; Rozenblat, 2004; Dicken, 2007), but it is not the only one in a multi-dimensional point of view.

In parallel to this, intense economic specialization creates groups of cities which are more-and-more interrelated (a tendency that is reinforced by policies of “poles of excellence” or “poles of competitiveness”). The most often quoted example of this specialization is the financial “global city” linking New York, London and Tokyo (Sassen, 1991). The financial capitals of the world as an ensemble attach themselves to each other either directly or indirectly by constituting specialized subgroups or/and geographical sub-groups.

Between these groups of highly interlinked cities, “relay” cities constitute “obligatory pathways” between the groups that can be depicted as “bridges” (fig.6). A telling example of this is constituted by the national capitals which host the national head-offices of the foreign subsidiaries of multinational corporations — head-offices which, in their turn, invest in businesses or establishments elsewhere in the host country. (The same model also functions at a continental level articulated around continental centers (Pumain, Rozenblat, 1993; Rugman, 2001; Alderson & Beckfield, 2004; Rozenblat, 2004 ; Rozenblat, Pumain, 2007). Meetings of employees are often located at these “intermediate” head-offices. At the level of cities, capitals perform the function of “bridgehead” for all the other national cities: for capital cities dramatize themselves by the expedient, for example, of providing a preferential location for representational offices (Brussels in Europe). Through this role of “display relay” for all the other cities of a given country, the capital thus concentrates a large proportion of relevant relationships both upstream and downstream: international and national. This position of “relay” or “*bridge*”, affords the capital better access to the whole network as well as increased control over information transfers (Rozenblat, Pumain, 2007). In a further dimension, the network positions of different cities which share similar attributes of “dependency” with regard to “relay” cities, display “equivalent” characteristics. Within each “sub-group” a marked hierarchy of cities’ levels of centrality in the network is maintained, while other cities remain altogether peripheral, being only attached indirectly to the network by virtue of their connection to participating cities. The recent reinforcement of the centrality of European national capitals is due to this process (Rozenblat, Cicille, 2003; Grasland, 2006; Rozenblat & al., 2008). To this end, one can wonder about the accumulation of social capital of cities in certain industries or business: what extent networks of certain activities, such as chemicals, can cause the emergence of other networks in relatively remote areas? To do this, we can refer to studies focusing on linkages between activities within certain cities (Hall, Pain, 2006).

The clusters and hierarchies create dynamics that can support, or move against the sustainable development of urban systems, which can be defined as the ability to renew and maintain geographical and social networks’ diversification. Thus, in a type of network "Clustered" components disconnected from the rest of the network can easily show that some groups of cities or clusters can be found relatively isolated from the rest of the urban system, without being able to overcome the decline of their core business. This process of fragmentation is detrimental to the isolated cluster that can not participate in the rest of the network, but also for the entire system loses its diversity. In contrast, one can assume that the cities of the same country do not develop all the same networks of expertise. This contributes to maintain a diversity of national urban network, provided that those mutual links between cities maintain a certain level of relations to enjoy this diversity.

It is of essential importance that our knowledge of all the properties of these networks of cities should be deepened in a multi-level perspective. Identifying, qualifying and measuring each level could make understand better globalization, because at each level, there are so many processes that allow the emergence of urban properties in which *“the whole becomes not only more than but very different from the sum of its parts”* (Anderson, 1972 in Lane, 2006).

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