

Draft-paper  
MULTIPOLAR REGIONALIZATION OF CITIES  
IN THE MULTINATIONAL FIRMS' NETWORKS

Prof. Céline Rozenblat, University of Lausanne, [Celine.rozenblat@unil.ch](mailto:Celine.rozenblat@unil.ch)

Dr. Faraz Zaidi, University of Lausanne, [farazahmed.zaidi@unil.ch](mailto:farazahmed.zaidi@unil.ch)

Antoine Bellwald, University of Lausanne, [Antoine.Bellwald@unil.ch](mailto:Antoine.Bellwald@unil.ch)

**ABSTRACT**

*This paper aims to identify the more cohesive regions of the world cities' graph without introducing any prior hierarchy in the premises of the approach, using a clustering method based on "modularity", which outline the high densities of the graph. A "Spin Glass" clustering procedure for weighted networks is used to identify different classes for industries or services aggregated by skill levels. We used a global database of the network of 1 million direct and indirect ownership links between the 800,000 subsidiaries of the top 3,000 multinational groups of the world in 2013, which are located within 1,205 metropolitan areas. Comparing the obtained partitions with benchmarks using Normalized Mutual Information (NMI), the classes of cities correspond partially to Free Trade Zones (FTZ) but exhibit interesting cross-continent patterns. A few cities change their classes according to the activity taken into account. They are discussed considering the strategies of multinational firms.*

**KEYWORDS**

MULTINATIONAL FIRMS, WORLD CITIES NETWORKS, COMPLEX SYSTEMS, CLUSTERING

## INTRODUCTION

In a multi-scaled geographic system, intense economic specialization creates groups of cities that are more and more interrelated despite distances. The most often quoted example of this phenomenon remains that of financial specialization, that has generated a “global city” centered on New York, London and Tokyo (Sassen, 1991). The other cities of the world are supposedly connected to this central system in a hierarchical pattern (Friedmann, 1986; Taylor, 2001; Brown et al., 2010) that admits a regionalization of large zones like USA, Pacific-Asia, Euro-Germany and Old Commonwealth (Taylor et al., 2002). Thus, this cities’ network is a complex combination of hierarchy and regionalization. However, the previous analyses on the regionalization of world cities integrated a very strong *a priori* assumption on hierarchy of cities in the globalization (Taylor et al., 2002, 2013; Derudder et al., 2003; Alderson & Beckfield, 2004; Alderson et al., 2010) and it seems that the regional pattern operating in parallel to hierarchy could be under-evaluated.

We assume that the network density of some cities’ communities reinforces the power of the highest “world cities” that constitute multi-poles bridging their internal community in the globalization. Free Trade Zones (FTZ) may structure these cohesions, but other geographic associations may also matter for the organization of global economic linkages (Ohmae, 1996; Dicken, 2003; Pomfret, 2007; Rugman et al., 2012). The question remains to identify the uneven geographical scales of these cities’ communities and to verify if they vary according to their level of skills (for industry and service).

We propose to identify dense networks of cities at the world scale through a large sample of multinational firms networks including all kinds of activities. We will focus on the network properties of cities’ communities defined by the different levels of skills included in services or industries. The hierarchical organization will first be discussed in relation to the regionalization of the global integration of cities (section 1); the hypotheses will be tested using a global database of networks of multinational firms between cities (section 2). We will evaluate their similarities with regional benchmarks and their mutual difference, underlying cities changing their class according to the activities (section 3). Then we will discuss the results (section 4).

### 1. REGIONALIZING CITIES BY MULTINATIONAL FIRMS

Previous studies often started from a classification of cities according to their role in multinational firms networks (Taylor et al., 2002, 2012; Derudder et al., 2003; Alderson & Beckfield, 2004; Alderson et al., 2010). These roles differ quite a lot according to the sample of activities taken into account (1.1). The combination of regionalization and hierarchy of cities, leads to the reflection on the place of the hierarchical concept in the regional vision of the world (1.2). On a methodological aspect, underlying protocols behind the clustering methods will orient the choice of network classification that we'll implement (1.3).

#### 1.1 Multiple globalizations

The cities’ system of control described by Sassen (1991, 2007), stresses a high spatial concentration of financial and specialized services in some cities organizing the whole industrial and service management. This development, coupled with the emergence of new corporate financial networks, led to a vertical disintegration (Storper, 1997; Scott, Storper, 2007) and increased the complexity of multinational firms’ networks (Powell, 1990). Firms coordinate their worldwide activities in a “global value chain” according to the complexity level of transactions (i.e. the ability to codify them) (Gereffi et al., 2005).

Thus, multinational companies represent “*networks within networks*” (Dicken, 2011, p.121) interacting with spatial preferences mixing national, regional and urban specializations in the development of their evolving “*global production network*” (Yeung, 2005; Coe et al., 2008; Dicken, 2011). Multinational firms to develop its sectorial activities, articulate financial and industrial activities, and different levels of intensive knowledge. Krätke (2014) identified distinctive worldwide organization of multinational firms’ networks for one service and two industrial sectors highlighting the main urban specializations, and

showing that world cities are the most diversified (Duranton, Puga, 2004; Pumain et al., 2012). In this perspective, we will consider that different groups of cities emerge according to levels of skill and to industry/services, revealing different spatial strategies of ‘multiple globalizations’ (Krätke, 2014, p.125) that firms articulate all together.

## **1.2 Regionalization and hierarchies of the world cities**

The economic regionalization of the world started with the vision of domination between countries. Myrdal (1957) and Wallerstein (1974) stated that the world could be classified roughly in three main zones: center, semi-periphery and periphery (Chase-Dunn, Rubinson, 1977; Chase-Dunn, 1998; Sanderson, 2005; Dezzani, Johansen, 2012). Very early, Hymer (1972) suggested that the pattern of regionalization among cities of the world is consistent with this hierarchy of countries. Alderson & Beckfield (2004) and Alderson et al. (2010) showed the evidence that cities of peripheral countries have less likelihood to have a high centrality score than cities of core countries. At the opposite side, Friedmann (1986) and Sassen (1991) assumed that the global network of cities would strongly differ from the system formed by countries, generating a distinctive urban system built-in the “new international division of labor”. This system comprises a “*complex spatial hierarchy*” that integrates both “*hierarchical and regional tendencies*” (Derudder et al., 2003; Liu et al., 2012; Taylor et al., 2013). The comparison Wall et al. (2011) undertook between the Alderson & Beckfield’s assumption and the Taylor et al.’s one, demonstrates the difficulty to distinguish hierarchy from regionalization in these complex networks.

## **1.3 Clustering methods and concepts of multi-polar regionalization**

Previous clustering approaches applied on the world cities’ networks often include premises influencing the “hierarchy” introduced at two levels of the analysis: in the data and in the methodology. For the data, the construction of the GaWC networks, which is based on a [firms X cities] matrix, builds the regionalization on the spatial overlaps of the different firms’ expansions (Taylor, 2001; Neal, 2012). When a clustering based on *cliquishness* was applied, all the central cities were in every cliques (Derudder, Taylor, 2005). For the methodological aspect, Alderson & Beckfield (2004) and Alderson et al. (2010) used *block modeling* taking into account the *structural equivalence* between “levels of cities” according to the level of their countries, that emphasized much more hierarchical aspects than regional ones.

*Complex System Sciences (CSS)*, developed methods that classify networks according to their “local densities” forming “communities” or “clusters” (Blatt et al., 1996; Girvan & Newman, 2001; Clauset et al., 2004; Newman, 2006, 2012; Reichardt, Bornholdt, 2006; Traag, Bruggemann, 2008; Sathik et al., 2011). Guimera et al. (2005) used a clustering algorithm for airplane flows, which revealed regionalization patterns of cities that were greatly consistent with those of main continents. Rozenblat et al. (2013) used a multi-level clustering, underlying the strategic role of main airlines companies in the regionalization of the world. CSS permits to handle the whole complexity of the networks, controlling better the methodology in order to not create some “artifact effect” of hierarchy.

## **2 Methodological premises to underline the multi-polar regionalization of world cities**

The network approach allows detecting the regionalization of world cities avoiding a strong *a priori* emphasis on the hierarchical aspect. A hierarchy will be revealed at the ultimate step of the analysis, when we will evaluate how communities are mutually interacting or internally organized.

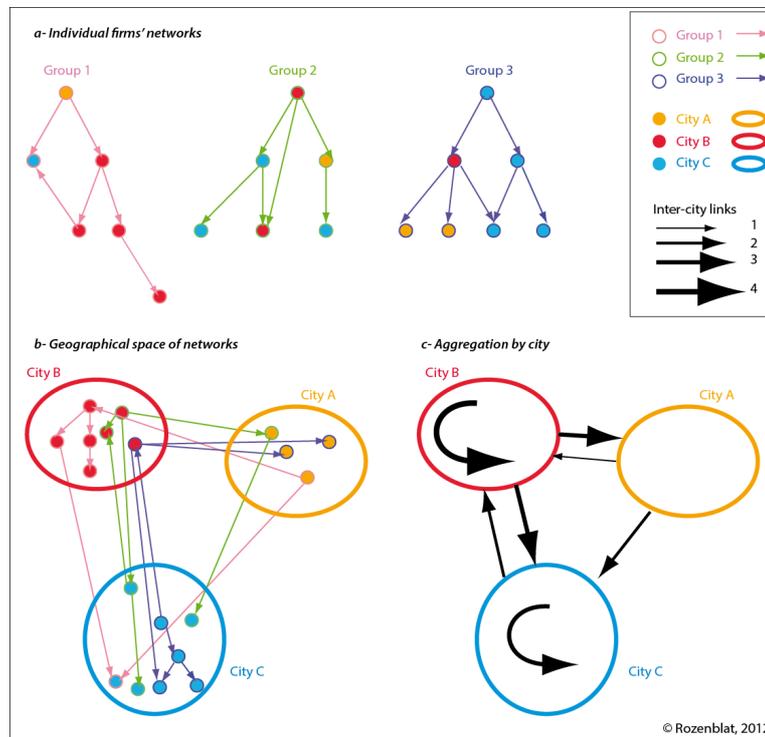
### **2.1 Building cities’ systems with the multinational firms’ networks**

To determine the positions of cities within such multinational firms’ networks, we first built a database starting from the top 3,000 worldwide company groups based on their turnover (Orbis, Bureau Van Dijk, 2013, Fig.1.a) with their direct and indirect 800,000 subsidiaries located all over the world. These subsidiaries are connected by 1 million financial links operating with successive steps. This set of networks is similar but larger to the one of Krätke (2014), Wall & Van der Knaap (2009) (but this latter is

restrained to a maximum of five subsidiary steps), and the ones of Alderson & Beckfield (2004) and Alderson et al.(2010) (one subsidiary step).

According to the locations of each of the 800.000 enterprises of the network (Fig.1.a), we then aggregated these groups networks at the city level (Fig.1.b and Fig.1.c).

Figure 1: Building data: from individual networks of firms to cities networks



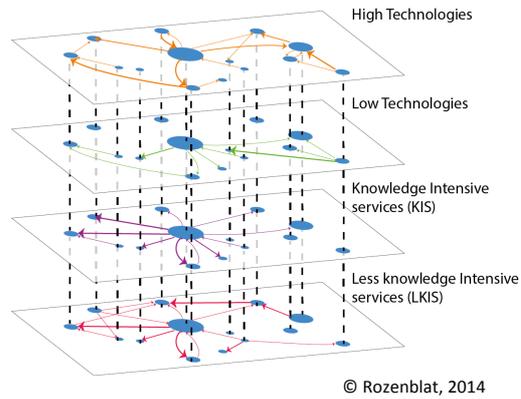
To aggregate the data within comparable “cities”, firms were precisely located into metropolitan areas delineated as ‘Large urban regions’ (LUR)<sup>1</sup>. Firms’ linkages were aggregated by LUR, using their origin and destination locations. This process yielded a matrix linking 1,205 cities from all over the world. These areas contain more than 85% of the overall links of the database<sup>2</sup>. The intercity relationships are oriented (from the owner city to the subsidiary city) and they are weighted by aggregating the oriented firms’ links by pairs of cities. We removed the intra-urban linkages, keeping only the inter-urban ones (Fig.1.c).

In addition to LUR, we might also consider different geographical territories, constituting benchmarks to compare to empirical clustering. The UNCTAD list of FTZ (2013) includes overlapping zones. We attributed a unique membership for each country according to the more relevant zones of economic openness for trade and subsidiaries. Two other benchmarks were built: large continents (7 continents according to the UN definition [2013]); geographical proximities (orthodromic distances).

## 2.2 Networks of multinational firms by level of knowledge of activities

In order to distinguish the globalization of production/service and of high/low level of skills, we adopted the OECD-EC classified activities (2009). This classification considers “as knowledge intensive if tertiary educated persons employed (according ISCED97, levels 5+6) represent more than 33% of the total employment in that activity” (OECD, 2009). OECD produced two distinct aggregations (high and low) for industry and for services. Activities were attributed to each single firm and its ascendant linkages (a group can encompass several activities). We thus created a four multi-layered network (Fig.2).

Figure 2: Multi-layered city networks



In order to compare these four layers, it was necessary to have the same number of cities and we kept only common ones. Thus the sample was reduced to 503 cities, which encompass more than 90% of the total linkages. Finally, after the location of firms in LUR, the removing of their intra-urban links and the reducing to the common cities between the layers, it remains 612,798 observed links of subsidiarity aggregated in the multilayer networks of 503 cities.

### 2.3 Clustering method

Complex System Science literature operates the detection of communities or clusters as the identification of groups of nodes densely connected to each other and sparsely connected to nodes from other clusters (Schaeffer, 2007). Numerous algorithms are classified according to partitional/hierarchical, divisive/agglomerative, fuzzy/hard properties (Jain *et al.*, 1999). Algorithms are also different if they can be applied on directed/undirected networks, weighted/non-weighted networks, or tree-like networks (Fortunato, 2009; Schaeffer, 2007). Another important distinction of recent network clustering algorithms has been the use of local or global optimization methods (Schaeffer, 2007). Local methods try to optimize some local function such as *modularity* (Newman, 2006), which is a measure of cohesiveness of groups of nodes (high intra-cluster edges vs. low inter-cluster edges). On the other hand, global optimization methods require the information of entire network structure (usually a lot of computation), and thus they are not suitable for large datasets. Local optimization methods are a good alternate as they provide efficient and relatively accurate clustering results to the cluster detection problem (Fortunato, 2009). Another popular categorization for the clustering algorithms is the use of dynamic processes such as *random walks*, *spin models* and *information propagation* (Pons & Latapy, 2004). These methods have resulted in a number of fast and accurate clustering algorithms (Fortunato, 2009).

We selected the appropriate methods for weighted and oriented data. We applied seven different clustering algorithms that are applicable on weighted networks and are belonging either to local or global optimization methods<sup>3</sup>. The comparison of the seven algorithms was undertaken in term of *modularity* (Girvan & Newman, 2001; Clauset *et al.*, 2004; Newman, 2012)<sup>4</sup> (Tab.1).

Table 1: Comparison of clustering algorithms

	Algorithm	Modularity
1	Edge Betweenness (Girvan & Newman, 2002)	0.02
2	Walk Trap (Pons and Latapy, 2004)	0.12
3	Info Map (Rosvall <i>et al.</i> , 2009)	0.29
4	Label Propagation (Raghavan <i>et al.</i> , 2007)	0.03
5	Fast Greedy (Newman, 2004)	0.31
6	Louvain (Blondel <i>et al.</i> , 2008)	0.31
7	Spin Glass (Reichardt and Bornholdt, 2006)	0.36

©Zaidi, Rozenblat, 2014; Source: ORBIS BvD, IGD-UNIL, Geodivercity, 2013

The best results in terms of modularity were obtained with the *Spin Glass* proposed by Reichardt and Bornholdt (2006). The Spin Glass algorithm<sup>5</sup> has several advantages such as it does not require the number of clusters to be known *a priori*. They are defined by the higher modularity index.

## 2.4 Comparing clustering communities' similarities

The resulting clusters will be compared with the benchmarks of regional areas. A number of cluster similarity measures have been proposed in the recent past (Lancichinetti & Fortunato, 2009) and we use the most widely accepted method called the Normalized Mutual Information (NMI) first proposed by Danon et al. (2005). Given two partitional structures of a network, calculation of NMI returns a value in the range between 0 and 1 where 1 suggests perfect similarity and values close to 0 indicate high dissimilarity. The condition to compare two networks requires the same number of nodes in the two networks, this is why we reduced the initial matrix to keep only the common nodes to every layers.

### 3- Clustering world cities according to multinational firms networks with different knowledge levels

The Spin Glass clustering applied in the same set of 503 cities, results of five different clustering outputs: one for the total network and four sub-networks for the two levels of skills of industry or services. The general comparisons of the different clustering results (3.1) will be explained by their proper characteristics (3.2). It will also permit to better interpret the main moves of cities from one cluster to another one depending to different activities' clustering (3.3).

#### 3.1 General comparisons of clustering results

The resulting five clustering can be compared according to their general characteristics (3.1.1), or they can be confronted to the benchmarks (3.1.2).

##### 3.1.1 General characteristics of clustering

Considering the modularity index (Tab.2), the main first result is that lower modularity indexes qualify the clustering outputs of the higher levels of skill (HIGH-TECH and KIS). This modularity index is not correlated to other properties of the network (Density, Hierarchy (Alpha-Power), Clustering Coefficient) and it does not exactly vary with the average internal links, because it measures both existing and non-existing edges.

Table 2: Application of Spin glass clustering by skill-levels of multinational firms' networks

	Nodes (cities) Number	Total Weighted edges	Density (total weighted edges/nodes number)	Alpha Power-law	Clustering Coefficient (Triads)	Modularity of Spin Glass clustering	Number of clusters	Average Internal Links (%)
<b>HIGH-TECH</b>	503	76,339	152	1.71	0.30	<b>0.31</b>	7	70%
<b>LOW-TECH</b>	503	55,653	111	1.73	0.28	<b>0.37</b>	9	83%
<b>KIS : KNOWLEDGE INTENSIVE SERVICES</b>	503	208,562	415	1.54	0.29	<b>0.33</b>	10	85%
<b>LKIS : LESS KNOWLEDGE INTENSIVE SERVICES</b>	503	146,938	292	1.66	0.30	<b>0.43</b>	11	93%
<b>TOTAL NETWORK</b>	503	612,798	1,218	1.32	0.33	<b>0.36</b>	10	71%

©Zaidi, Rozenblat, 2014

Low modularity indexes for high skills networks mean that these communities of cities are less "cohesive" and exchange more between clusters. The result makes sense: firms with the highest skills activities would search for rare competences in other communities, while firms with more banal low skills activities, located closer to the markets, would organize the networks with stronger routines creating more "cohesive" cities' communities.

The number of classes is higher for industrial activities clustering (7 for HIGH-TECH and 9 for LOW-TECH) than for the services ones (10 for KIS and 11 for LKIS), meaning a more partitioned organization of groups of cities but that are not necessary more cohesive: LOW-TECH with less numerous clusters than KIS, has got a higher modularity index. The explanation of this subtle difference between the number of clusters and their modularity needs the overlook to other aspects and thus will be explained later.

##### 3.1.2 Comparison of the clustering results with Benchmarks

The comparison with benchmarks informs on the proximity of the activities' networks of cities and regional or spatial patterns (Tab.3).

Table 3: NMI comparison of clustering of cities with several benchmarks

Clustering of cities according to firms' networks	(1) TOTAL Clustering	(2) UN 7 REGIONS	(3) Free Trade Zones	(4) UN 22 SUB-REGIONS	(5) Countries	(6) Distance-Based Clustering
HIGH-TECH	0.67	0.62	0.63	0.59	0.54	0.54
LOW-TECH	0.77	0.61	0.62	0.60	0.65	0.52
KIS	0.86	0.54	0.58	0.61	0.77	0.52
LKIS	0.82	0.52	0.63	0.63	0.77	0.52
AVERAGE	0.78	0.55	0.60	0.60	0.70	0.52
TOTAL clustering		0.49	0.50	0.51	0.61	0.55

©Zaidi, Rozenblat, 2014

The interurban networks of KIS and LKIS remain highly organized around national urban systems compared to the industries. The principal reason is the necessity for services to be aware of national rules and laws that maintains a strong organization of enterprises at the national level (Sassen, 2007). At the opposite side, interurban networks of HIGH-TECH have only half similitude to the national delineations, meaning an extended international organization based on highly specialized cities networks. These networks remain largely (two-thirds) inside their respective FTZ.

Around 50% similarities are observed between every clustering with the distance-based clustering, which is lower than the similarities between FTZ. It is an opposite result that Barigozzi et al. (2011) found for international trade between countries where distance more matter than the belonging to FTZ, but it is not found at the same scale because here distances are also measured between cities inside countries. It could mean that the distance is not the only criteria to interurban interaction and one can assume that urban hierarchy of each country also still matter a lot.

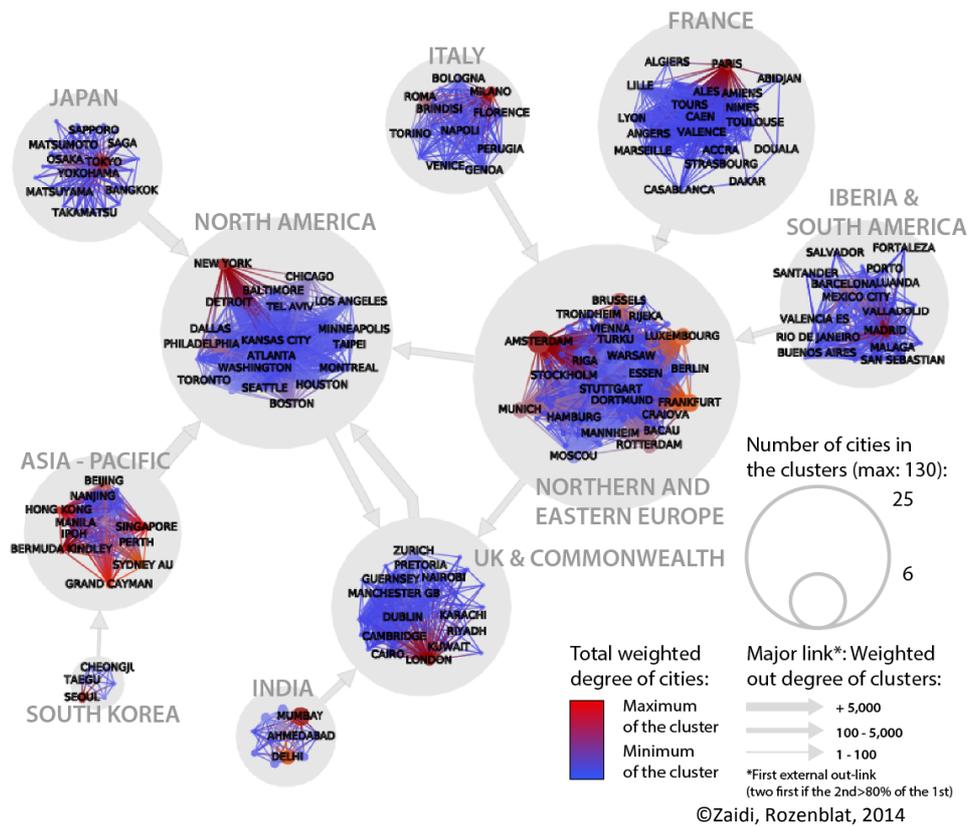
### 3.2 Description of the clustering results

One can better understand the differences between communities analyzing how they are organized in the whole network (3.2.1), or for the different activities (3.2.2), specifying the main cities moving from a group to another one (3.2.3).

#### 3.2.1 Cities' clustering on the whole network

The clustering on the whole network results of ten clusters, that one can identify by the dominant regions or countries of their cities and by the main links between them (Fig.3).

Figure 3: Clustering of cities' networks with all activities



- North America encompasses 93 North American cities including Canadian ones, but also Taipei, Tel Aviv and Veracruz. This class is led by New-York concentrating 16.5% of the internal links of the class, followed by Philadelphia (9.6%), Chicago (5.8%) and Boston (5.3%).
- Asia - Pacific is dominated by Chinese cities (Hong-Kong: 17%, Beijing: 13.4%, Shanghai: 8.6%), but also regroup South Eastern Asian cities (Singapore, Kuala Lumpur) and Australian cities (Sydney: 6%, Melbourne: 3%, Perth and Brisbane). This class also comprises tax havens as Grand Cayman (8%), Bermuda Kindley (10%) or Tortola (Virgin Islands).
- Japan includes 31 cities: all Japanese ones plus Bangkok, Jakarta and Maastricht. The class is highly polarized by Tokyo concentrating 45% of the class and Osaka (25%).
- South Korea: includes all the 6 main South Korean cities. Similarly to Tokyo, Seoul polarizes 46% of the internal links, while Pusan 20%.
- Indian community of 10 cities highlights the network cohesion of this sub-continental country, including also Mauritius and Colombo (Sri Lanka). Mumbai polarizes this group with 36% of the internal links, followed by Delhi with 23% and Chennai 8%.

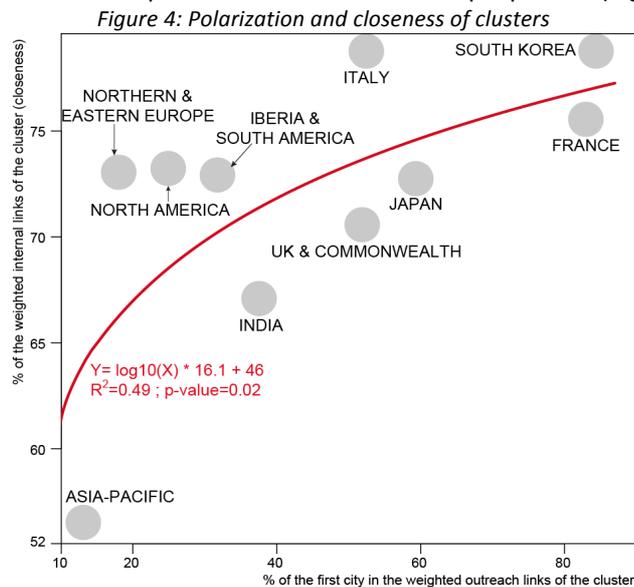
European cities are clustered in 5 communities:

- Northern and Eastern Europe comprises 130 cities including Northern to Eastern Europe until Russian and Turkish cities. While Amsterdam has got the most important relations (8.3%), it does not polarize it so much. Many other cities interact equally as Vienna, Frankfurt, Brussels, Moscow, Luxembourg, Rotterdam, Hamburg, Munich, Warsaw each concentrating internal links between 5% and 3%.
- UK & Commonwealth regroups 49 cities, and is polarized by London (35% of total internal links). This community is the most spatially spread one. Beyond UK and Irish cities, it encompasses Swiss ones, but also cities from Middle East (Abu Dhabi, Kuwait, Riyadh, Amman until Karachi), cities from Africa (the four main South African cities) and capitals of Africa such as Cairo, Lagos, Nairobi or Gaborone.
- Iberia and South America: Madrid polarize 35% of the internal links, Barcelona 14% followed by Bilbao, Lisbon, Sao Paulo, Seville, Valencia, Rio de Janeiro, Mexico City, Santiago de Chile and Buenos Aires.
- The French group is polarized by Paris (39% of the total internal links) but includes as well ancient French colonies in Africa (Tunis, Algiers, Douala, Dakar) or French speaking capitals (Accra).

- Another national network includes the 35 Italian cities of our dataset without any exception, and encompasses also Lugano, the Swiss Ticino city, which is Italian speaking.

The system of dominant relations between clusters (Fig.3) highlights the centrality of three clusters: the North American cities' cluster, the Northern and Eastern Europe and the UK & Commonwealth ones. These three clusters polarize most of the links of firms' investments going to or coming from the other clusters of cities (74% of the total weighted linkages between clusters). Compared to the different clustering results proposed by Derruder et al. (2003) and completed by Liu et al. (2013), or the ones calculated by Alderson & Beckfield (2004) or Alderson et al. (2010), the Spin Glass clustering disconnects the main global cities, integrating them in their proper regional or cultural groups.

In this regional organization, the more global cities concentrate a large part of the investments controlled by or attracted by the cluster's cities to/from outside the cluster. However, the clusters are very diverse in their concentration of networks in one or few cities, or in their capacity to encompass a large part of the cities' networks. We plotted these two internal properties (Fig.4).



It reveals a correlation between the polarization of the clusters' outreach linkages and its level of closeness: more a dominant city concentrates the outreach links of its proper cluster less the other cities have some external linkages. Some clusters are very polarized around one city if one considers either internal or external links (Paris for France, Seoul for South Korea, Tokyo for Japan, Milano for Italy, London for UK and Commonwealth). Other clusters are multi-polarized system with a similar level of closeness of Japan or UK (North America, Northern and Eastern Europe, Iberian and South American).

The cities less integrated inside clusters by the share of their internal linkages (Tab.4), are either some anomalies of geographical membership: Maastricht in the Japanese cluster or Mauritius within the Indian one; either cities that could make bridge between clusters: South American cities that are also partly associated with North America and with Europe, Swiss cities that are mostly included in the UK & Commonwealth cluster but also integrated with all the other cluster, or tax heaven places like Bermuda Kindley.

*Table 4: Cities with the lowest integration in clusters (less than 30%)*

Large Urban Region	COUNTRY	CLUSTER	linkages of the city inside the cluster (in % of the total linkages of the city)
MAASTRICHT	NETHERLANDS	JAPAN	11.6
MANILA	PHILIPPINES	ASIA / PACIFIC	13.9
JAKARTA	INDONESIA	JAPAN	14.6
BANGKOK	THAILAND	JAPAN	18.1
QUITO	ECUADOR	IBERIA & SOUTH AMERICA	19.1
CARACAS	VENEZUELA	IBERIA & SOUTH AMERICA	19.3
ACCRA	GHANA	NORTH AMERICA	21.3
COLOMBO	SRI LANKA	INDIA	21.4

GUADALAJARA	MEXICO	IBERIA & SOUTH AMERICA	21.4
ZURICH	SWITZERLAND	UK & COMMONWEALTH	22.6
MEXICO CITY	MEXICO	IBERIA & SOUTH AMERICA	23.7
PANAMA CITY	PANAMA	IBERIA & SOUTH AMERICA	24.8
MAURITIUS	MAURITIUS	INDIA	25.9
SAO PAULO	BRAZIL	IBERIA & SOUTH AMERICA	26.5
CAIRO	EGYPT	UK & COMMONWEALTH	26.7
SAO JOSE DOS CAMPOS	BRAZIL	IBERIA & SOUTH AMERICA	26.9
BERMUDA KINDLEY	BERMUDA	ASIA / PACIFIC	27.1
GENEVA	SWITZERLAND	UK & COMMONWEALTH	28.4
BUENOS AIRES	ARGENTINA	IBERIA & SOUTH AMERICA	29.8

### 3.2.2 Cities' clustering for activities' networks

The clustering implemented with the same methodology in the different activities' networks, allows understanding better the general network. We'll not describe so deeply each of them, but we'll focus on their main resemblances and differences. The main differentiation on the clusters' composition opposes industrial networks to services networks (Tab.5). Especially HIGH-TECH clusters are the most different to the others. The two closest clustering are the ones of services.

Table 5: NMI comparison of clustering of cities by activities' skill level

	HIGH-TECH	LOW-TECH	KNOWLEDGE INTENSIVE SERVICES	LESS KNOWLEDGE INTENSIVE SERVICES
HIGH-TECH	1			
LOW-TECH	0.72	1		
KNOWLEDGE INTENSIVE SERVICES	0.65	0.67	1	
LESS KNOWLEDGE INTENSIVE SERVICES	0.56	0.65	0.78	1

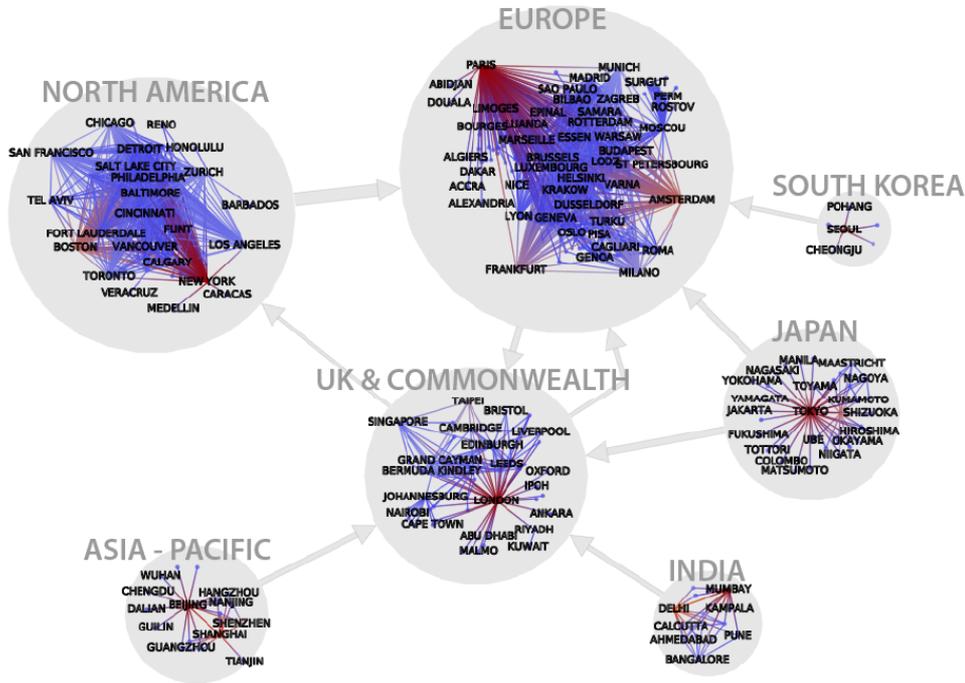
©Zaidi, Rozenblat, 2014

Some clusters keep a quite similar set of cities: North America, Japan, UK & Commonwealth, Asia-Pacific constitute quite stable clusters (Fig.5). For industrial clusters, Europe is a whole community beside UK (except for the Italian cluster for LOW-TECH), while for services, Europe is much more fragmented in several clusters. At the opposite, for industrial sectors, Korean cities are apart, while for services they are encompassed into Asia-Pacific cluster.

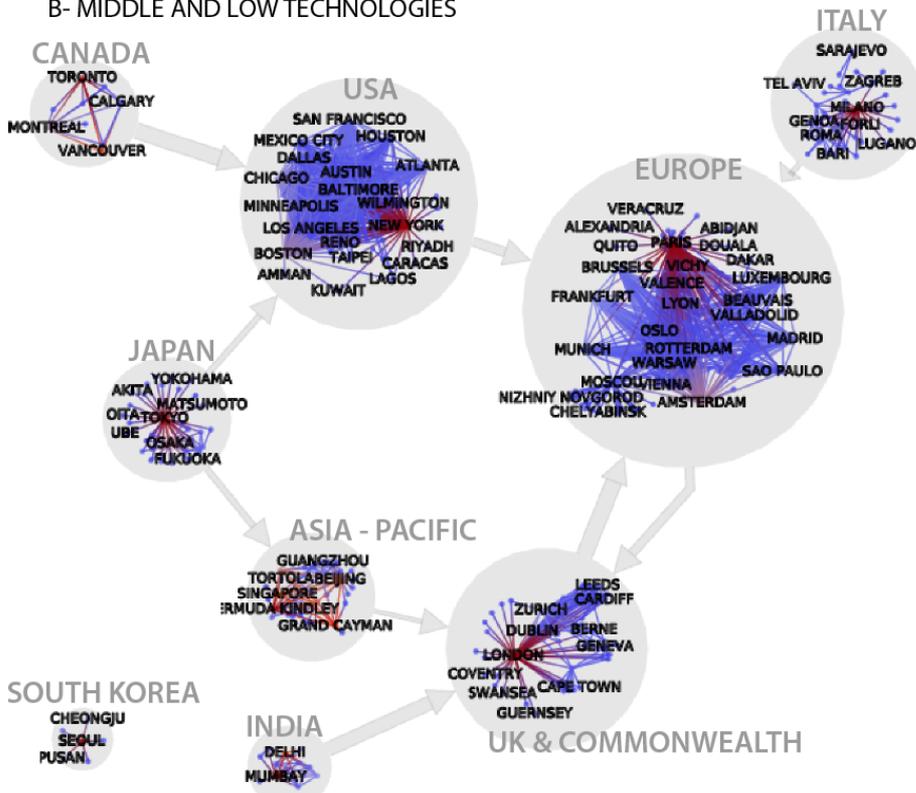
The weights of the three dominant clusters North America, Europe and UK-Commonwealth, in terms of the share of weighted links in the whole networks, vary from 85% for higher skills activities of industries or services, to 75% for lower skills activities. It means that the higher skills activities networks are more concentrated inside these dominant cities' clusters, which make sense in the perspective of the diffusion of innovation where the most skills activities remain closer to the central cities' regions.

Figure 5: Clustering of cities' networks by skill levels of activities

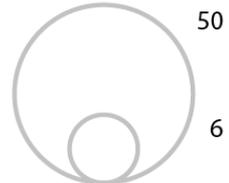
A- HIGH TECHNOLOGIES



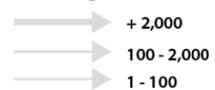
B- MIDDLE AND LOW TECHNOLOGIES



Number of cities in the clusters (max: 275):



Major link\*: Weighted out degree of clusters:



\*First external out-link (two first if the 2nd > 80% of the 1st)

Total weighted degree of cities:

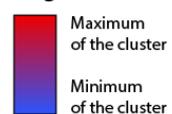
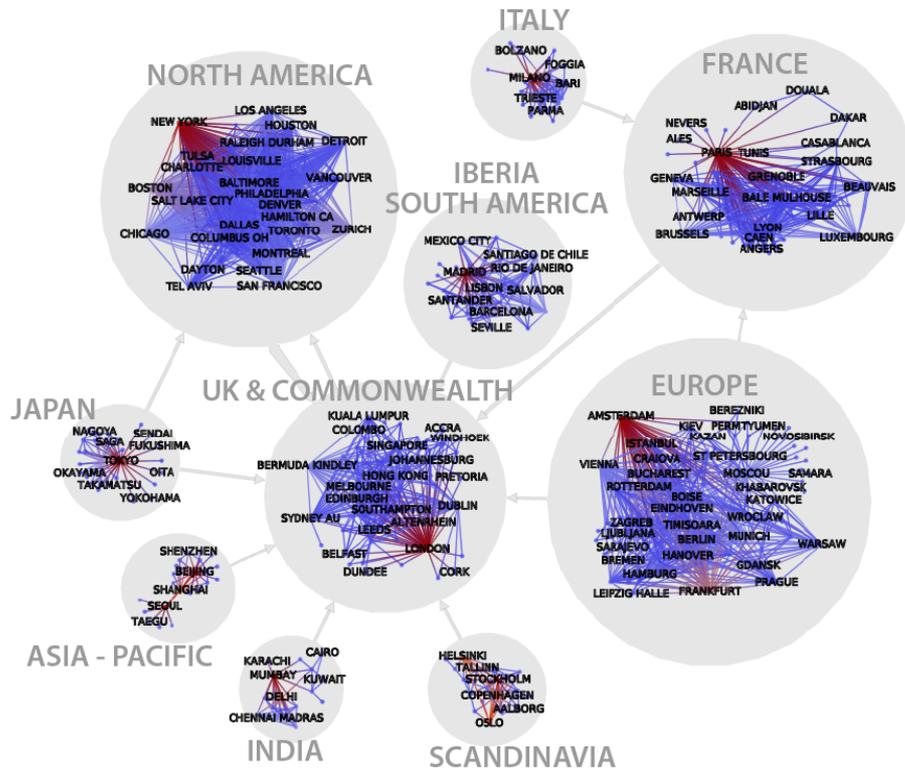
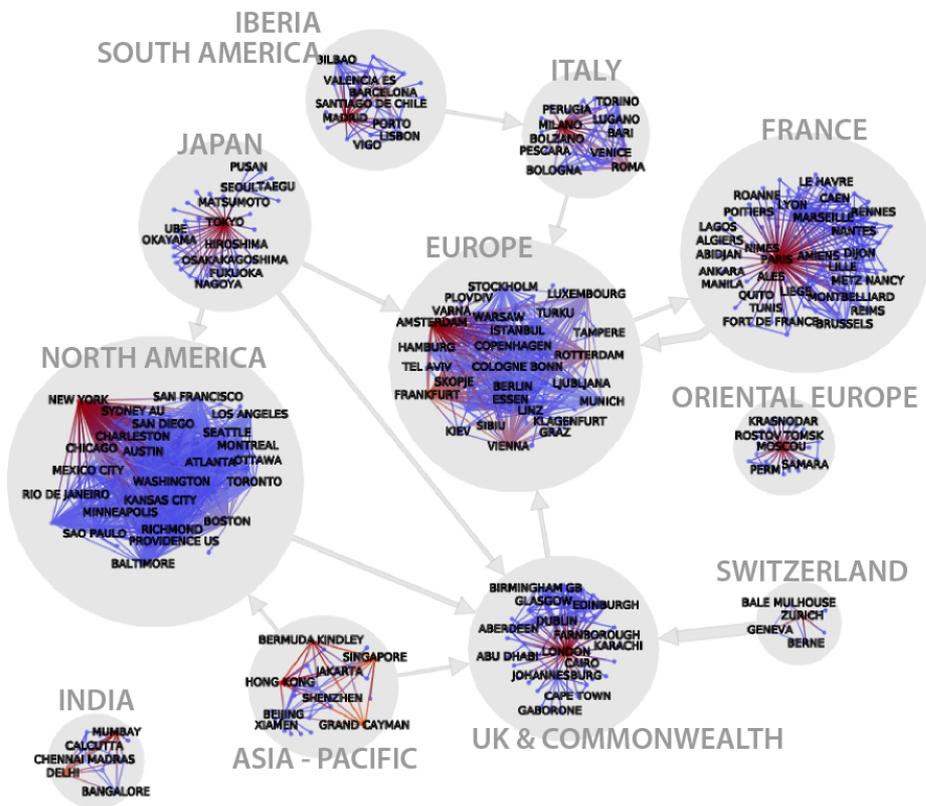


Figure 5 (continuing): Clustering of cities' networks by skill levels of activities

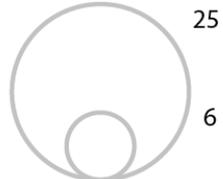
C- KNOWLEDGE INTENSIVE SERVICES



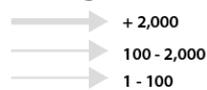
D- LESS KNOWLEDGE INTENSIVE SERVICES



Number of cities in the clusters (max: 108):

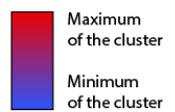


Maximum total weighted out degree of clusters\*:



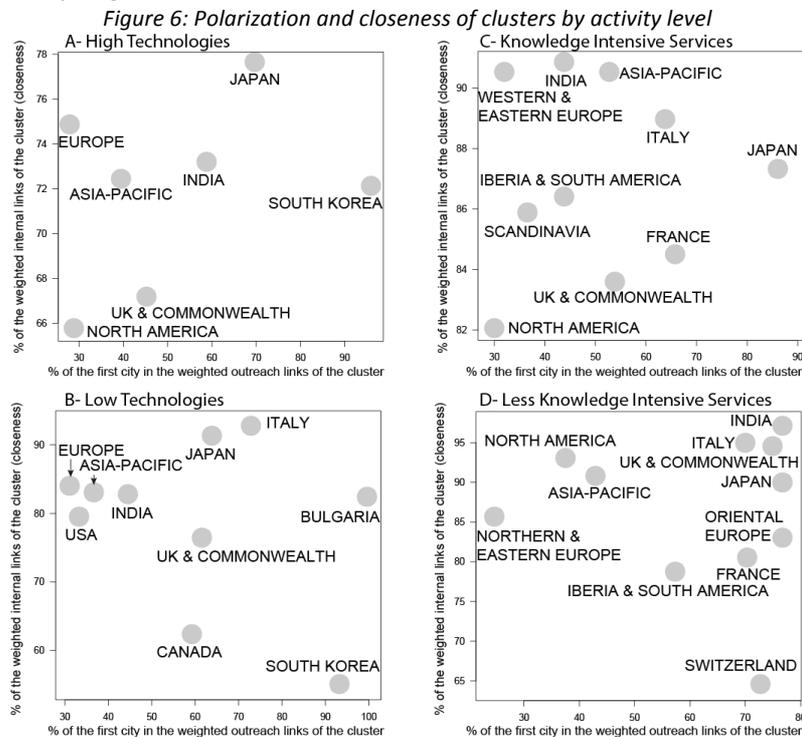
\*First external out-link (two first if the 2nd > 80% of the 1st)

Total weighted degree of cities:



Regarding the mutual positions between clusters, the UK & Commonwealth cluster is very central among the clusters for the KIS (Fig.5.C). For the Less KIS, UK & Commonwealth share the central place with North America and Europe developing major relations with other clusters forming like satellites. It is in LOW-TECH, that the centrality is the more shared between different clusters, forming like a loop of clusters.

In order to synthetize the internal organization of the clusters, we plotted once again the polarization of the clusters' outreach linkages in the highest city and the average level of closeness of the cities of the clusters for each activity (Fig.6).



There is no correlation between the two properties of the clusters with a high variation of internal situations of the clusters depending to the activities. The unique regularity is visible for the North American and the UK & Commonwealth clusters remaining the two more open clusters for the more skilled activities. In these two cases, they are strongly central and encompass a large number of cities. But most of the time, clusters are recomposed and transformed in four main distinct ways:

- Firstly, the clusters are divided in several sub-clusters between skilled and less skilled activities;
- Secondly, cities shifting from a cluster to another one can change radically the organization of the clusters. When France is aggregated to Europe, Paris becomes the highest city. Iberian & South American cities that form a cluster for services, are divided between North America and Europe for industry. Thus these two clusters are transformed by their new cities' compositions;
- Thirdly, the stable clusters are generally dominated by the same city but there are exceptions like the Asian-Pacific cluster, mostly dominated by Beijing except Hong-Kong for LKIS internal links, Shanghai for HIGH-TECH outreached links, and Seoul for KIS outreached links.
- Fourthly, the role of secondary cities matters a lot depending to the activity: For HIGH-TECH, Taipei is the second city after London in the UK & Commonwealth cluster, encompassing more than 15% of the total outreach links of the cluster; Osaka second in the Japanese cluster, is particularly strong for HIGH-TECH, encompassing more than 20% of the external linkages thanks to its headquarters of the *Panasonic company*.

Thus the properties of the clusters are widely influenced by their cities' composition and by the balance between them. The shifts from a cluster to another one, and from an internal position to a different one, according to activities underline the differences between the different aspects of globalization.

When a city remain in the same cluster whatever the activity, one can consider that it has a strong and deep attachment to its cluster. At the opposite, a city moving from a cluster to another one reflects different forms of integration into the globalization. While clusters change themselves from one activity to another one, the moves of cities must be considered without taking into account the major re-compositions of the clusters. This is why we aggregated clusters by their main composition in 5 major stable communities:

- Europe: France, Italy, Spain, Europe, GB & Commonwealth;
- North America: USA and Canada;
- Asia: Asian cities and Korea;
- Japan;
- India.

Among the 503 cities that we implemented, 428 (85%) remain in the same major stable community whatever the activity taken into account. Beside, 75 cities move from a cluster to another one, following sometimes some common patterns.

The European community seems quite powerful to integrate cities in high skilled activities, having common characteristics of tax haven (Bermuda Kindley, Hong-Kong, Grand Cayman, Singapore, Kuala Lumpur or Virgin Islands), or energetic based economy (Lagos in Nigeria, Almaty and Astana in Kazakhstan).

On another hand, the North American cities have privileged high skills links with South American or with some very high skilled cities like Zurich or Tel Aviv. The Japanese intercity networks are also powerful to integrate for HIGH and LOW-TECH Asian cities and even European ones. Indian cities networks integrate Middle East cities like Cairo, Alexandria, Abu Dhabi, Riyadh, Amman and Kuwait for KIS.

#### **4. Discussion on the multi-polar organization of the world cities' system**

Overall, the analysis encourage the belief of the world cities' network, as a "multipolar" system because of these very cohesive clusters forming cities communities interacting more together than with other ones. The shift to a multipolar vision of the world cities corresponds to a wider change in the comprehension of the feedback processes maintaining or transforming the distribution of powers between cities and especially forming the predominance of few of them. The classical core and peripheries system is replaced by a two levels system considering cores and peripheries both between and within clusters. This double hierarchy between and inside clusters expresses a networking process running by steps where investments of firms often enters into a cluster of cities by the top cities in order to reach other cities where they diffuse their investments. FTZ are of high importance in this stepwise process. It participates to the feedback maintenance and reinforcement of the global strength of the central cities of each cluster, but it also reinforces the internal cohesion of each cluster. In this sense, global linkages between clusters and local ones, inside clusters are intertwined.

Most of the dominant cities of the central clusters correspond to the ones that are classically mentioned as "world cities": London, New York, Tokyo, Paris, Amsterdam... The regularity of their centrality in the service or industrial sector was already mentioned by the correlation between the two networks underlined by Wall and Van der Knaap (2011). However the level of their cluster domination and their cluster's hierarchical characteristics specify the core/hinterland results of the previous studies made on the regionalization and hierarchy of the world cities' networks (Taylor et al., 2002, 2013; Derudder et al., 2003; Alderson & Beckfield, 2004; Alderson et al., 2010). These specifications can be stressed in three main results.

Firstly, the "classical world cities" do not have the same level of domination in their own clusters, which induce different forms of globalizations:

- On one hand, the group of cities, that highly dominates their cluster, concentrate more than the half of the total out-linkages of their cluster: London, Tokyo, Paris but also Milano and Seoul. These cities concentrate a high intermediary capacity of decision, with a central role in the globalization for the other cities of their own sub-system. The best example for this kind of worldwide role is London, encompassing a global control on numerous cities all over the world. Other cities like Tokyo, Paris, Milano and Seoul control primarily their national urban system.

- On another hand, New-York for North American cities, Amsterdam for Northern and Eastern Europe, Beijing for Asian-Pacific cities belong to more distributed systems where many cities exchange firms' linkages directly with cities of other clusters. Firms' practices exploit their combined networked resources that are linked with outside the clusters.

Secondly, the multipolar system of cities is not functioning at the same scale all over the world. The fact that some continental, inter-continental or national scales appear at the same score of modularity expresses a wide range of levels of international openness/cohesion of the cities by countries and continents. Japanese, Korean, Indian, Chinese, French, Italian cities and even Swiss cities for LKIS, remain with a high level of national mutual interdependencies. These networks between national cities produce the same level of dependencies existing in much wider communities like North America or UK-Commonwealth. On this aspect, the Spin Glass clustering does not verify the conclusion that, "*clusters with low average connectivity tend to be more regionally restricted in membership*" (Taylor et al., 2002, p.885).

Thirdly, the inter-cluster level not only reveals the mutual position of dominant cities, but also the organization of the whole sub-systems. The quite stable core centrality of North American, European and UK-commonwealth cities according to the different activities is simultaneous of cities' movements: national cities create their own clusters like for Swiss cities or Oriental European ones for LKIS, South Korean cities for Industry or Canadian cities for LOW-TECH; Cities moving from a cluster to another one have a particular position in the globalization process because they are mostly oriented to different groups of cities according to the activity. Cities of Asia, Australia, South America, Zürich and Tel Aviv are in between several communities. Therefore they constitute specialized or regional pivots in the expansion of the multinational firms strategies, hosting their regional or specialized centers of industry or of services.

## **Conclusion**

The systematic method of clustering borrowed from Complex System Sciences allows ordering the regional processes and the hierarchical ones between cities. It reveals that cities are pooled in cohesive communities that are unevenly hierarchized around one dominant city. Firms articulate in their own networks these skills / less skills activities and industrial and services functions in their global value chain and in their production value chain. In this sense, the multi-polarized cities' networks represent the main sub-systems of cities resources for high/low skills of industrial and services activities that are practices by the worldwide firms' networks. The orientation of the networks from headquarters to subsidiaries determines the role of cities inside the clusters.

In terms of policy, this new vision offers to each city an overview of its main interdependencies. Then this analysis could help them to build a strategy both inside the cluster: to increase the density of links inside the cluster that could be an easy way of specialization; and outside the cluster by targeting to specific cities according to which groups of cities they would like to reach. Governments of the cities that are always dominated by the same city are in general well aware of that dependency and generally try to diminish it, fostering other orientations of linkages. At the opposite side, the government of cities that are in between several groups could use this property to strengthen the position of their cities by trying to attract new firms from targeted activities and from specific cities. Of course, the main global cities (London, New-York, Paris, Tokyo, Amsterdam) will probably continue to attract all the links from everywhere, by a "natural" process of preferential attachment that reproduce their attractiveness and their concentration of power. However, the resilience of the cities' system being in the diversity of their interdependencies, this evolution has to be followed in order to evaluate the diversity level progression of the whole system and of each city dependency inside this system.

## **BIBLIOGRAPHY:**

Alderson, A. S., & Beckfield, J. (2004). Power and Position in the World City System. *American Journal of Sociology*, 109(4), 811-851.

- Alderson, A. S., Beckfield, J., & Sprague-Jones, J. (2010). Intercity Relations and Globalisation: The Evolution of the Global Urban Hierarchy, 1981--2007. *Urban Studies*, 47(9), 1899-1923.
- Barigozzi, M., Fagiolo, G., & Mangioni, G. (2011). Identifying the community structure of the international-trade multi-network. *Physica A: Statistical Mechanics and its Applications*, 390(11), 2051-2066.
- Blatt, M., Wiseman, S., & Domany, E. (1996). Superparamagnetic Clustering of Data. *Physical Review Letters*, 76(18), 3251-3254.
- Blondel, V. D.; Guillaume, J.-L.; Lambiotte, R. & Lefebvre, E. (2008). Fast unfolding of communities in large networks. *J. Stat. Mech.*, IOP Publishing
- Bollen, K. A. and Appold S. J. (1993) National Industrial Structure and the Global System, *American Sociological Review* 58(2): 283-301.
- Bordo, M. D., Taylor, A. M., & Williamson, J. G. (2005). *Globalization in Historical Perspective*. University of Chicago Press.
- Bornschieer, V., Chase-Dunn, C., & Rubinson, R. (1978). Cross-national evidence of the effects of foreign investment and aid on economic growth and inequality: A survey of findings and a reanalysis. *American Journal of Sociology*, 651-683.
- Boyd, J. P., Mahutga, M. C., & Smith, D. A. (2013). Measuring Centrality and Power Recursively in the World City Network: A Reply to Neal. *Urban Studies*, 50(8), 1641-1647.
- Brown, E., Derudder, B., Parnreiter, C., Pelupessy, W., Taylor, P. J., & Witlox, F. (2010). World City Networks and Global Commodity Chains: towards a world-systems' integration. *Global Networks*, 10(1), 12-34
- Chase-Dunn C. and Rubinson R. (1977). "Toward a structural perspective on the world-system." *Politics and Society* 7, 4:453-76.
- Chase-Dunn, C. (1998) *Global Formation: Structures of the World-Economy*. Oxford: Blackwell.
- Clauset, A., Newman, M. E. J., & Moore, C. (2004). Finding community structure in very large networks. *arXiv:cond-mat/0408187*.
- Coe, N. M., Dicken, P., Hess, M., & Yeung, H. W.-C. (2010). Making connections: Global Production Networks and World City Networks. *Global Networks*, 10(1), 138-149.
- Danon L., Diaz-Guilera A., Duch J. & Arenas A. (2005). Comparing community structure identification *Journal of Statistical Mechanics: Theory and Experiment*, IOP Publishing, P09008
- Derudder, B., & Taylor, P. (2005). The cliquishness of world cities. *Global Networks*, 5(1), 71-91.
- Derudder B., Taylor P.J, Witlox F., & Catalano G. (2003). Hierarchical tendencies and regional patterns in the world city network: a global urban analysis of 234 cities. *Regional Studies*, 37(9), 875-886.
- Dezzani, R. J., & Johansen, H. (2012). The role of foreign direct investment as a structural development indicator of the hierarchical world economy. *Environment and Planning A*, 44(3), 580 - 604.
- Dicken, P. (2003). *Global Shift: Reshaping the Global Economic Map in the 21st Century*. SAGE.
- Dicken, P. (2011). *Global Shift, Sixth Edition: Mapping the Changing Contours of the World Economy*. Guilford Press.
- Duranton, G., Puga, D. (2004). Micro-foundations of urban agglomeration economies, In: Henderson, J.V., Thisse, J.-F. (Eds.), *Handbook of Regional and Urban Economics*, vol. 4. Elsevier, Amsterdam, pp. 2063-2117
- Friedmann J. (1986). The World City Hypothesis. *Development and change*, vol.17, 1, pp.69-84, re-printed in Knox P., Taylor P.J. (eds.), 1995, *World Cities in a world system*, Cambridge University Press, 317-331.
- Fortunato S (2009) Community detection in graphs *Physics Reports*, Elsevier, 2010, 486, 75-174.
- Gereffi G. Humphrey J. Sturgeon T. J. (2005). The Governance of Global Value Chains. *Review of International Political Economy*, Vol. 12, 1, 78-104
- Girvan P., Newman M. (2002), Community structure in social and biological networks. *Proc Natl Acad Sci USA* 99:8271{8276}.
- Gereffi, G., Humphrey, J., & Sturgeon, T. (2005). The governance of global value chains. *Review of international political economy*, 12(1), 78-104.
- Guimerà, R., Mossa, S., Turtschi, A., & Amaral, L. a. N. (2005). The worldwide air transportation network: Anomalous centrality, community structure, and cities' global roles. *Proceedings of the National Academy of Sciences*, 102(22), 7794-7799.
- Hymer S. (1972). The multinational corporation and the law of uneven development in J. Bagwati (ed.) *Economic and world order*, Ney York, MacMillan
- Jain AK, Murty MN, Flynn PJ (1999) Data clustering: a review. *ACM Comput Surv* 31(3):264{323}.
- Krätke, S. (2014). How manufacturing industries connect cities across the world: extending research on 'multiple globalizations'. *Global Networks*, 14(2), 121-147.
- Lancichinetti, A. & Fortunato, S. (2009). Community detection algorithms: a comparative analysis, *Physical review E*, APS, 80, 056117
- Liu, X., Derudder, B., & Liu, Y. (2013). Regional geographies of intercity corporate networks: The use of exponential random graph models to assess regional network-formation. *Papers in Regional Science*.
- Mahutga, M. C., Xiulian Ma, Smith, D. A., & Timberlake, M. (2010). Economic Globalisation and the Structure of the World City System: The Case of Airline Passenger Data. *Urban Studies*, 47(9), 1925-1947.
- Myrdal, G. (1957). *Economic Theory and Underdeveloped Regions*, London, Duckworth. pp. 66-86.
- Neal, Z. (2012). Structural Determinism in the Interlocking World City Network. *Geographical Analysis*, 44(2), 162-170.
- Newman, M. E. J. (2006). Modularity and Community Structure in Networks. *Proceedings of the National Academy of Sciences*, 103(23), 8577-8582.
- Newman, M. E. J. (2012). Communities, modules and large-scale structure in networks. *Nature Physics*, 8(1), 25-31.

- OECD (2009). OSLO MANUAL: GUIDELINES FOR COLLECTING AND INTERPRETING INNOVATION DATA, OECD/European Communities, <http://www.oecd-ilibrary.org>
- Ohmae K. (1996). *The end of the nation state: the rise of regional economies*. Free Press Paperbacks.
- Pomfret R. (2007). Is Regionalism an Increasing Feature of the World Economy? *World Economy*, 30(6), 923–947.
- Pons, P. & Latapy, M. (2004). Computing communities in large networks using random walks J. of Graph Alg. and App. bf, 2004, 10, 284-293
- Powell, W. (1990). Neither market nor hierarchy: network forms of organization. *Research in organizational behavior*, 12, 295–336.
- Pumain D., J. Portugali, H. Meyer, E. Stolk, & E. Tan. (2012). Urban systems dynamics, urban growth and scaling laws : the question of ergodicity. In *Complexity Theories of Cities Have Come of Age : An overview with implications to urban planning and design* (Springer, p. 93-106). Berlin.
- Raghavan, U.N., Albert, R. and Kumara, S. (2007). Near linear time algorithm to detect community structures in large-scale networks. *Phys Rev E* 76, 036106.
- Reichardt, J., & Bornholdt, S. (2006). Statistical mechanics of community detection. *Physical Review E*, 74(1), 016110.
- RosvallM., Axelsson D. & C. T. Bergstrom (2009). The map equation, *Eur. Phys. J. Special Topics* 178, 13
- Rozenblat, C., Melançon, G., Bourqui, R., & Auber, D. (2013). Comparing Multilevel Clustering Methods on Weighted Graphs: The Case of Worldwide Air Passenger Traffic 2000–2004. In Rozenblat, Melançon (eds), *Methods for Multilevel Analysis and Visualisation of Geographical Networks* (p. 141–154). Springer.
- Rozenblat C., Pumain D. (2006). Firm linkages, innovation and the evolution of urban systems. in P.J. Taylor et al (ed.) *Cities in Globalization*. Practices, policies and theories. Routledge. 130-156
- Rugman, A. M., Oh, C. H., & Lim, D. S. K. (2012). The regional and global competitiveness of multinational firms. *Journal of the Academy of Marketing Science*, 40(2), 218-235.
- Sanderson, S. K. (2005). World-Systems Analysis After Thirty Years: Should It Rest in Peace? *International Journal of Comparative Sociology*, 46(3), 179-213.
- Sassen (1991). *The Global City : New York, London, Tokyo*. Princeton University Press.
- Sassen S. (2007). *A sociology of globalization*, W.W. Norton, 2007 - 308 p.
- Sathik, M., Senthamarai Kannan, K., & Rasheed, A. (2011). Comparative analysis of community discovery methods in social networks. *International Journal of Computer Applications*, 14(8), 27–31.
- Scott, A. J., & Storper, M. (2007). Regions, globalization, development. *Regional studies*, 41(S1), S191–S205.
- Schaeffer S.E. (2007) Graph clustering. *Computer Science Review* 1(1):27{64}
- Smith, D. A., & Timberlake, M. (1995). Conceptualising and mapping the structure of the world system's city system. *Urban Studies*, 32(2), 287–302.
- Storper, M. (1997). *The regional world: territorial development in a global economy*. Guilford Press.
- Taylor P.J. (2001). Specification of the World City Network. *Geographical analysis*, 33 (2), 181-194.
- Taylor, P. J., Catalano, G., & Walker, D. R. F. (2002). Exploratory Analysis of the World City Network. *Urban Studies*, 39(13), 2377-2394.
- Taylor, P. J., Derudder, B., Hoyler, M., & Ni, P. (2013). New regional geographies of the world as practised by leading advanced producer service firms in 2010. *Transactions of the Institute of British Geographers*, 38(3), 497–511.
- Traag, V. A., & Bruggeman, J. (2008). Community detection in networks with positive and negative links. *Arxiv preprint arXiv:0811.2329*.
- United Nations (2013). Composition of macro geographical (continental) regions, geographical sub-regions, and selected economic and other groupings <https://unstats.un.org/unsd/methods/m49/m49regin.htm>
- Wall, R. S., Burger, M. J., & van der Knaap, G. A. (2011). The geography of global corporate networks: the poor, the rich, and the happy few countries. *Environment and Planning A*, 43(4), 904-927.
- Wall, R. S., van der Knaap, G. A. (2011). Sectorial Differentiation and Network Structure Within Contemporary Worldwide Corporate Networks. *Economic Geography*, 87(3), 267–308.
- Wallerstein, I. (1974). The Rise and Future Demise of the World Capitalist System: Concepts for Comparative Analysis. *Comparative Studies in Society and History*, 16(04), 387-415.
- Williamson, O. E. (1975). *Markets and hierarchies : analysis and antitrust implications ; a study in the economics on interna l organization*. New York [u.a.]: Free Press [u.a.].
- Yeung, H. W. (2005). Rethinking relational economic geography. *Transactions of the Institute of British Geographers*, 30(1), 37–51.

<sup>1</sup>'Large urban areas' were delineated according for Europe to the last ESPON definition 2012, to Metropolitan Statistical Areas for the USA and Canada and the equivalent for the main cities across the globe based on National delineations when available. This work was done by the collaboration of two teams: *IGD-UNIL Lausanne* and *Geodiversity* in Paris.

<sup>2</sup>Although we have a larger sample of subsidiaries, we obtain less numerous urban units than Alderson & Beckfield (2004, 2010), with 3,692 cities for 2000 and 6,308 cities for the period 1981-2007, because we considered the definition of urban areas to be much broader (in Europe, large urban areas are defined according to commuting flows; for instance, Wolfsburg is associated with Braunschweig and Vevey as well as Lausanne are aggregated with Geneva).

<sup>3</sup>The seven tested algorithms are :

- *Edge Betweenness* (Girvan and Newman, 2002): The Edge Betweenness clustering algorithm is a global algorithm based on the calculation of geodesic distances. The edges lying in between all pairs of nodes are removed repeatedly to break the entire network in disconnected components thus giving us different communities.
- *WalkTrap* (Pons and Latapy, 2004): The Walk Trap algorithm is a global optimization using a dynamic process of random walks to calculate distances between different nodes, based on which nodes are grouped together to form clusters using Ward's Method. Modularity is used to select the best partition of the resulting dendrogram.
- *Info Map* (Rosvall *et al.*, 2009): Info Map algorithm is another global optimization algorithm belonging to the category of dynamic process as it also uses random walks where the idea is to compress the information of this dynamic process minimizing description length of the random walk to obtain clusters.
- *Label Propagation* (Raghavan *et al.*, 2007): The Label Propagation algorithm belongs to the category of local algorithms using a dynamic process of majority voting. Randomly unique labels are assigned to individual nodes which are subsequently updated using a majority voting in the neighborhood of the vertex, which in turn results in the detection of communities in the network.
- *Fast Greedy* (Newman, 2004): The Fast Greedy algorithm is a local clustering algorithm, which aggregates nodes closer to each other based on the optimization of an objective function called Modularity (discussed in detail below).
- *Louvain* (Blondel *et al.*, 2008): The Louvain algorithm is also another local optimization algorithm based on modularity where the aggregated nodes are replaced by a new node in the graph, thus simplifying the original graph every time a new cluster is formed. This method results in a different and faster clustering algorithm.
- *Spin Glass* (Reichardt and Bornholdt, 2006): The Spin Glass clustering algorithm is based on the idea that if Potts spin variables are assigned to the vertices of a network, and the interactions are between neighboring spins, structural clusters can be found from spin alignment of the system. Thus a local optimization takes place to align neighboring nodes and resulting in communities.

<sup>4</sup>Mathematically, Modularity is defined as follows (Girvan & Newman, 2001; Clauset *et al.*, 2004; Newman, 2012):

$$Q = \frac{1}{2m} \times \sum_{vw} \left[ A_{vw} - \frac{k_v k_w}{2m} \right] \delta(c_v, c_w)$$

where

$$A_{vw} = \begin{cases} 1 & \text{is an edge of the graph between nodes } v \text{ and } w \text{ (1 if } v \text{ and } w \text{ are connected; 0 otherwise)} \\ 0 & \end{cases}$$

$$k_v = \sum_w A_{vw} \text{ is the degree of } v \text{ and } k_w = \sum_v A_{vw} \text{ is the degree of } w$$

$$m = \frac{1}{2} \sum_{vw} A_{vw} \text{ is the total number of possible non-oriented edges in the graph}$$

$\delta(c_v, c_w)$  is the community function for  $v$  and  $w$ : 1 if  $v$  and  $w$  are in the same community; 0 otherwise.

<sup>5</sup>The spin glass algorithm models the cluster detection problem as finding the ground state of an infinite ranged Potts spin glass. Based on the foundations of this model (which is a "*Hamiltonian of a (disordered) Potts model*" [Newman, 2012]), a group of physicists generalized this Hamiltonian Potts model, developing the "*spin glass clustering methods*" usually applied to the physics of materials (inhomogeneous ferromagnetic materials), which often require oriented and weighted interaction models (Reichardt, Bornholdt, 2006). The spin glass methods use the Hamiltonian Potts model  $H[\{s\}]$  of a partition  $s$  that measures the distance between the nodes according to their direct or indirect links to the networks  $J_{vw}$  and based on their membership within the classes being partitioned  $\delta(s_v, s_w)$ :

$$H[\{s\}] = - \sum_{vw} J_{vw} \delta(s_v, s_w)$$

The spin glass model takes into account the existing links as well as the missing links inside and outside each class. Then,  $H[\{s\}]$  is calculated for 4 cases (Reichardt, Bornholdt, 2006):

---


$$H[\{s\}] = - \underbrace{\sum_{v \neq w} a_{vw} J_{vw} \delta(s_v, s_w)}_{\text{internal\_links}} + \underbrace{\sum_{v \neq w} b_{vw} (1 - J_{vw}) \delta(s_v, s_w)}_{\text{internal\_non\_links}} + \underbrace{\sum_{v \neq w} c_{vw} J_{vw} (1 - \delta(s_v, s_w))}_{\text{external\_links}} + \underbrace{\sum_{v \neq w} d_{vw} (1 - J_{vw}) (1 - \delta(s_v, s_w))}_{\text{external\_non\_links}}$$

Thus, the model measures both intra-community and inter-community links. It also evaluates the “*temperature*”, which is minimized when the network is far to randomly graph (Blatt et al., 1996). It calculates different partitions (called spin states), maximizing their modularity and minimizing the temperature.