A Restatement of Walras' Theories of Capitalization and Money

Aldo Montesano[*]

Introduction

Walras introduces the general equilibrium theory in successive stages, the following one including the preceding one. These stages are: the theory of exchange of two commodities for each other (Part II of the Elements); the theory of exchange of several commodities for one another (Part III); the theory of production (Part IV); the theory of capitalization and credit (Part V); and the theory of circulation and money (Part VI).[1]

The analysis of the exchange and production of fixed capital goods was presented by Walras in the second edition of his Elements. This theory, though misunderstood by some readers, is perfectly consistent.[2] A version of it, slightly different from the original Walrasian version, is given in Section 3: a kind of securities, which is issued and exchanged, is introduced in place of the Walrasian fictitious good, the perpetual net income.

Most of the discussion, however, will be devoted to the theory of money, which is the most controversial part of the Walrasian system. Walras introduced in the successive editions of his Elements three different versions of the theory of money.[3] In the first edition the Walrasian theory of money consists of a transaction equation, which is a completely analogous formulation to the one developed later by Fisher. In the second and third editions the notion of *circulation à desservir* is substituted by the notion of *encaisse désirée*, of desired cash balance, which is a formulation analogous to the one developed later by Marshall.[4] In the fourth edition (and in the definitive one) Walras presents a very elaborate theory of money which is connected, unlike the preceding ones, to the remaining part of his theoretical building, because of the link between money and circulating capital goods (consumer goods and raw materials). The money services, which give an indirect utility to money, consist of obtaining the availability of circulating capital goods. In other words, the demanded quantity of money depends on the marginal utilities (and, for producers, by the coefficients) of the availability services of the circulating capital goods. Walras' theory of money as been less successful than his theories of exchange and of production. The theory of money has been neglected even if some economists have appreciated it, and

[*] Bocconi University, Milan. This paper is a revision of preceding versions (Montesano, 1986 and 1991).

[1] Obviously, data introduced in a subsequent stage contribute to determine variables introduced in the preceding stages. For instance, the coefficients of production (introduced in Part IV) contribute to determine the prices of consumer goods (introduced in Parts II and III). On the contrary, Negishi (1977, pp. 602-03 and 611) believes that money does not affect variables except the general level of prices, because money is introduced in Part IV, after the determination of the relative prices in the preceding Parts. This argument is similar to Nogaro's (1906, pp. 687-88). However, relative prices do not depend on money if money is a veil.

[2] A debate on the Walrasian theory of capitalization took place among Italian economists in the sixties (Montesano, 1970 and 1971). The Walrasian theory of capitalization was discussed, among others, by Jaffé (1953), Floss (1957), Morishima (1960), Collard (1973), Walker (1984) and van Daal (1998).

[3] These three versions are proposed respectively by Walras (1874), Walras (1889 and 1896) and Walras (1900 and 1926). An outline is given by Jaffé (Walras, 1954, pp. 600-02) and Porta (1980, pp. 18-34).

[4] It has been discussed whether this theory anticipates the Cash-Balance Approach, as Marget (1931) declares, or it provides only a Cash-Balance Equation, as Patinkin (1965, pp. 542-46) asserts (in this case a demand for money determined by its marginal utility would not exist).

criticisms, developments and discussions have not been lacking.[5] I believe that Walras' theory of money (of course, the last version) contains a very interesting core. This theory provides the general equilibrium approach to the analysis of money as the medium of exchange, which is its only possible use in the world without friction, uncertainty and illusion which Walras has idealized. In other words, Walras' theory concerns the pure transaction demand for money. This approach can be a starting point to more realistic analyses, which are obtained by introducing friction (in particular, transaction costs) and uncertainty (thus considering the precautionary and speculative demand for money). Walras' theory of money, however, is tarnished by some ambiguities and inconsistencies. Moreover, the literary account of the theory does not always correspond to its mathematical formulation.[6] The primary aim of this paper is not to comment Walras' theory of money, but to restate it through a formulation avoiding the original ambiguities and inconsistencies. Obviously, any new formulation (among many possible)[7] implies an alteration of the original theory. The restatement proposed by this paper retains, I believe, the most significant part of Walras' theory, showing, moreover, its importance mainly in view of further analyses.

I. Some observations on Walras' theories of capitalization and money

The Walrasian theories of exchange and production do not take into account the exchange and production of fixed capital goods and the inventories of products and money. Their introduction and analysis are respectively the object of the theory of capitalization and credit and of the theory of circulation and money, which can be very briefly summarized as follows. The fixed capital goods are owned only in view of the income obtained by selling their services. Assuming expectations of stationary prices and that the capital goods produced in the period under examination can be used only from the following period, a uniform rate of net return results for all fixed capital goods, which are consequently perfect substitutes with infinitely elastic demands. Savings are determined, according to consumer preferences, as a function of all prices, comprising the rate of net return. The theory of circulation and money concerns the services of availability given by the inventories of commodities and by money. The services of availability consist of using commodities before the end of the period of production under examination. Formally, inventories and money are demanded by consumers according to the utility of their services of availability and by entrepreneurs according to their coefficients of production. Holding inventories and money implies a cost, which consists of giving up the income attainable by holding fixed capital goods or securities. In equilibrium, the value of the availability services equals the cost of holding inventories. Taking into account that the original formulation of Walras' theory is affected by inconsistencies, it is appropriate to discuss it in some detail, before proposing a coherent new formulation. Walras' theory of circulation and money is based on some hypotheses which must be highlighted. Starting from those concerning real goods, the first hypothesis excludes any uncertainty on the prices and the dates of the exchanges performed during the period under examination (Walras, 1954, p. 317). This

[5] Pareto and his followers disregarded Walras' theory of money. Schumpeter (1954, pp. 1020-26 and 1082-83) approved of it. Criticisms were given by Nogaro (1906), Del Vecchio (1909), Hicks (1933) and Patinkin (1965). Aupetit's theory is a development of Walras' theory. Walras' theory of money was discussed, among others, by Marget (1931 and 1935), Rosenstein-Rodan (1936), James and Lecoq (1961), Kuenne (1961 and 1963), Collard (1966), Morishima (1977), Negishi (1977), Jaffé (1980), Hall (1983), Walker (1991), Bridel (1997) and Rebeyrol (1998).

[6] On this aspect, for instance, Hall (1983).

[7] An example is given by the so-called neowalrasian theory of money, which is declared neowalrasian only because money is considered in the general equilibrium scheme (albeit a macroeconomic one) and it is included in the utility function. The origin of this theory can be attributed to Hicks (1935), its more significant elaboration is provided by Patinkin (1965). Reformulations nearer to Walras' theory are those proposed by Aupetit (1901), Kuenne (1963) and Morishima (1977).

hypothesis will be indicated as the hypothesis of *certainty in the period*. The second hypothesis excludes that the goods produced during the period under consideration can give the services of availability during the same period: these services can be supplied only in the successive period (Walras, 1954, p. 319). This hypothesis will be indicated as the hypothesis of *invariability of the circulating capital in the period*. The third relevant hypothesis, which is implicitly assumed by Walras in his theory of capitalization, requires *expectations of stationary prices*.[8]

The hypothesis of certainty in the period and of expectations of stationary prices imply that inventories and money are not demanded for a speculative or precautionary motive, i.e. with regard to possible changes in prices or possible delays in the availability of commodities. Inventories are held (and are useful and productive) since there are asynchronies within the period under examination.[9] That is, not only does production require time (for outputs to be available after the correspondent inputs are introduced), but also products are used by consumers and producers after they have been made available by production.

Consequently, inventories are not constant during the period but vary according to asynchronies. Moreover, for any agent, any commodity, and money there is at least one instant, within the period under examination, where the correspondent inventory is zero. In fact no-one will hold a permanent inventory, which is costly and useless (because of the hypothesis of certainty in the period).[10]

The hypothesis of invariability of the circulating capital in the period means that the endowment of circulating capital goods is a datum (as well as that of fixed capital goods), thus unchangeable within the period. This hypothesis implies that inventories are used once and only once by consumers and producers. I.e., the owners of inventories consume directly or sell to other agents the commodities they have, which are wholly consumed during the period under examination. The products of this period can be stored and give their services of availability only in the following period. Thus, the remuneration of the services of availability supplied by inventories must be referred to the whole length of the period, independently from the particular instants in which commodities are given and returned.

One problem, which has been considered neither by Walras nor by scholars who have commented on his theory, is the fact that the goods consumed during the period derive both from inventories and from current production. To these two groups of commodities, however, different values correspond, since the value of goods from inventories also includes the cost of the services of availability (in addition to the cost of production). This situation is inconsistent with the condition requiring only one price for any commodity. Thus, we must require not only that products cannot give the services of availability in the same period of their production, but also that they cannot give in this period consumer or productive services.[11] Consequently, only the commodities existing in the initial inventories can be used during the period, while the commodities produced in the period can be used in the following period. This hypothesis (not

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[8] Walras' theory concerns the temporary equilibrium, i.e., time is divided in periods connected to each other also through the presence of durable goods. An equilibrium corresponds to each period (Montesano, 1970-71, pp. 710-12, Donzelli, 1986, pp. 264-68, and Witteloostujin-Maks, 1988).

[9] Marget (1935, pp. 160-61), Kuenne (1963, pp. 288 ff.).

[10] The hypothesis of expectations of stationary prices excludes not only permanent inventories of circulating capital goods in production but also of circulating capital goods the production of which is unprofitable or impossible. In fact, such goods would be stored without being wholly consumed within the period under examination only if the expected price for the following period is at least $(1+i)$ times the actual price, where i is the interest rate. This possibility would be admitted if the hypothesis of expectations of stationary prices excludes the consumer goods and the raw materials which are not produced in the period. The consideration of inventories of these goods will be disregarded for the sake of simplicity.

[11] Moreover, the assumption that products can be consumed in the same period of production, while they cannot be stored and supply the services of availability, seems to be excessively strong.

assumed by Walras, but determined by the logic of consistency), for which *produced goods are not available before the end of the period,* implies the hypothesis of invariability of the circulating capital in the period.

Examining now particularly the role of money, the principal point of Walras' theory seems to be the hypothesis that consumers and producers can maintain purchasing power during the period only through money (Walras, 1954, pp. 316-317), i.e. there is *impossibility of non-monetary exchanges during the period.* Moreover, Walras assumes that money can be lent (at the current interest rate). Consequently, agents must use money for their purchases; the quantity of money is related to current payments; and the cost (or the proceeds) due to the payment of interests is proportional for any agent to the excess (or the lack) of payments over the endowment of money. It is appropriate in this scheme to assume that agents can lend money only at the beginning of the period, i.e., *the impossibility of loans after the beginning of the period*. Otherwise, every agent would hold money in the instant when he gets cash and he would lend this sum soon after asking for reimbursement the moment he purchases something. Thus, a very small quantity of money would be sufficient for a large amount of exchanges and the value of money, the quantity of which is not nil by assumption, would be negligible.[12] With the hypothesis of impossibility of loans after the beginning of the period, agents who have a sufficient quantity of money in their endowments continue to hold a quantity equal to the payments envisaged for the period under examination and they lend the outstanding quantity. On the contrary, agents who have no money in their endowments (or have an insufficient quantity) borrow the quantity of money they need for their payments. In this way money supplies a service of availability which is identical to that supplied by inventories of goods, taking into account also that the hypothesis of certainty in the period determines the partition of the quantity of money of every agent in his purchases.

In Walras' theory, however, the demand for the availability services in money concerns commodities produced and consumed within the period, since the commodities existing in agents' endowments supply the availability services in kind. If we assume as proposed above, that products are not available before the end of the period, then money has no role, unless we modify Walras original description, as will be done in the following Section.

Neither does Walras consider that the availability services in money and those in kind are perfect substitutes for all agents and, thus, that there do not exist different functions of demand for them.[13]

Moreover, Walras introduces the relationships of the theory of circulation and money as if inventories and money could directly supply their services of availability with their presence: i.e. he does not treat explicitly the sale of stored goods to consumers and producers and the utilization of money in exchanges, which is the circulation of goods and money.[14]

Finally, Walras does not introduce the process of determination of the quantity of money between successive periods of time. i.e., not only is the quantity of money a datum for the period

[12] This reasoning is the basis of Hicks' criticism (1933, pp. 446-48) of Walras' theory of money. Also Rosenstein-Rodan (1936, pp. 271-72) follows this reasoning, while hinted at this earlier Knight (1921, pp. 193-94 footnote). The above assumption has been used by Kuenne (1963, pp. 298-99).
Here money is paper money. The use of credit money (bank deposits) changes the description of the problem: this possibility will be briefly analyzed in Section 4.
[13] This is Del Vecchio's principal criticism (1909, pp. 269-72) of Walras' theory. Also Hall (1983, p. 250).
[14] The need to consider money as the medium of exchange is emphasized by some economists, particularly by Clower (for instance, Clower 1967), against the neowalrasian theory of money. However, neither Clower (1967) nor his follower Howitt (1973) take into account that Walras (1954, pp. 316-317) assumes the impossibility of non-monetary exchanges during the period.

under examination, but it is also constant in time if there are not exogenous changes. This constancy disagrees with the hypothesis of expectations of stationary prices in the case of progressive state (considered by Walras the most appropriate for his theory), i.e., when the quantities of goods and services grow.[15]

II. A new formulation of Walras' theories of capitalization and money

After having indicated the foundations and the inconsistencies of Walras' theory of circulation and money it is time to propose a new formulation which avoids those inconsistencies and includes all Walrasian theories. The following hypotheses are assumed:

a) *certainty in the period*;
b) *expectations of stationary prices*;
c) *unavailability of produced goods before the end of the period*;
d) *impossibility of non-monetary exchanges during the period* (these exchanges concern the circulating capital goods, while all other transactions –concerning fixed capital goods and their services, the goods produced in the period, securities and their coupons, interests and principals of loans– are settled at the end of the period under examination with monetary payments only for the balance of any agent);
e) *impossibility of loans after the beginning of the period*.

In this formulation two other groups of agents are introduced together with consumers and producers. They are the *owners of fixed capital goods* (including, in this formulation, also land and the old fixed capital goods, which were produced in past periods and are not produced in the period under consideration) and the *owners of circulating capital goods*. The owners of fixed capital goods sell the services of land and other fixed capital goods relative to the period under examination; they buy these goods, which are delivered and paid for at the end of the period; they finance their purchases by issuing securities. These decisions are taken in order to maximize expected profit (equal to the difference between the net income of capital goods and the coupons of issued securities). The owners of circulating capital goods (i.e., goods which can be used by consumers and producers only once) sell these goods during the period under examination against money; at the end of the period they buy the circulating capital goods produced in the period; they finance their purchases by issuing money. These decisions are taken in order to maximize expected profit (equal to the difference between the proceeds of sales and the cost of purchases).

Consequently, consumers, whose endowments include all the money (which has been issued at the end of the period before the one under examination), retain the amount of money necessary for purchasing consumer goods during the period and they lend the rest to producers, who use it for their purchases of productive goods.

Let us now examine the relationships with respect to the aforementioned four groups of agents.

1) Owners of fixed capital goods

The endowments of the owners of fixed capital goods are composed of assets represented by a vector $\bar{\boldsymbol{K}} = (\bar{K}^1, ..., \bar{K}^{n_K})$ of quantities of goods bought in the past and of liabilities represented by securities $\bar{B}$. (Securities are perpetual and measured by their coupons: thus the

[15] Aupetit (1901, pp. 149-51) is aware of the need to introduce an endogenous change in the quantity of money. However, Aupetit's theory refers to a metallic money produced under the same conditions as other capital goods. But in Walras' theory paper money is considered, the cost of production of which is zero.

unitary security yields one unit of money at the end of every future period). If the prices of fixed capital goods are indicated with vector $\boldsymbol{p}_{kk} = (p_{kk}{}^{1}, ..., p_{kk}{}^{n_K})$, we have the relationship

$$\text{(1)} \qquad \boldsymbol{p}_{kk}\overline{\boldsymbol{K}} = p_B z\overline{B}$$

where $\overline{B}$ is the quantity of securities issued in the past, p_B is the market value of a unitary security, and *z* is a factor by which the quantity $\overline{B}$ is multiplied in order to obtain the equality between the values of assets and liabilities. In other words, since in each issue the value of securities equals that of assets, if some changes in prices occur between successive periods, then the value of securities issued in the past would change (assuming that the emerging profits are distributed to the holders of securities). This change could be attributed to the coupons and to the price of old securities (if we prefer to keep their quantity constant) or to the quantity of old securities (if we prefer to have an identical price for all securities). This latter way is chosen in the present description. Thus, the variable $z \geq 0$ is introduced and the quantity $\overline{B}$ of securities issued in the past becomes $z\overline{B}$ in the period under examination.

The owners of fixed capital goods sell the services relative to the period at the prices $\boldsymbol{p}_k = (p_k{}^{1}, ..., p_k{}^{n_k})$, which are paid at the end of the period, when the coupons $z\overline{B}$ are also paid. The profit relative to the period under examination of the owners of fixed capital goods is

$$\Pi_k = (\boldsymbol{p}_k - \hat{\boldsymbol{v}}\ \boldsymbol{p}_{kk})\overline{\boldsymbol{K}} - z\overline{B}$$

where $\hat{\boldsymbol{v}}$ is a diagonal matrix representing the coefficients of depreciation and risk (called $\mu + \nu$ by Walras). Since the owners of fixed capital goods do not use the services, their supply is totally inelastic.

The owners of fixed capital goods invest (i.e., they demand fixed capital goods) by maximizing the present value of expected profits, defined by the difference between the proceeds of the sales of services and the costs represented by the coupons of the securities issued for financing the purchases of the corresponding capital goods. Representing with $\boldsymbol{d}_{kk}, s_{BB}$ the quantities of demanded capital goods and of offered securities of an investor, with index *e* the expected values, and with index *t* the future periods, the present value of expected profits is $\pi_k^e = \sum_{t=1}^{\infty} \upsilon_t ((\boldsymbol{p}_{k,t}^e - \hat{\boldsymbol{v}}\ \boldsymbol{p}_{kk,t}^e)\boldsymbol{d}_{kk} - s_{BB})$, where υ_t is the factor of discount for time *t*, under the finance constraint $p_B s_{BB} = \boldsymbol{p}_{kk}\boldsymbol{d}_{kk}$. Consequently,

$$\pi_k^e = \sum_{t=1}^{\infty} \upsilon_t (\boldsymbol{p}_{k,t}^e - \frac{1}{p_B} - \hat{\boldsymbol{v}}\ \boldsymbol{p}_{kk,t}^e)\boldsymbol{d}_{kk}$$

Since the hypothesis of expectations of stationary prices requires $\boldsymbol{p}_{k,t}^e = \boldsymbol{p}_k$ and $\boldsymbol{p}_{kk,t}^e = \boldsymbol{p}_{kk}$ and the condition of perfect competition requires $\pi_k^e = 0$, we obtain

$$\text{(2)} \qquad \boldsymbol{p}_k = (\hat{\boldsymbol{v}} + \frac{1}{p_B}\boldsymbol{I})\,\boldsymbol{p}_{kk}$$

i.e., rates of net return of fixed capital goods (which are $\frac{p_k^i}{p_{kk}^i} - v^i$ for $i = 1,\ldots, n_k$) and of securities (which is $\frac{1}{p_B}$) are all equal.[16] Consequently, at prices satisfying relationship (2), capital goods are perfect substitutes and their demand is infinitely elastic.[17] Equations (1) and (2) imply that the current profit Π_k of the owners of fixed capital goods is zero.

The last equation concerning the owners of fixed capital goods is their total finance constraint

$$\boldsymbol{p}_{kk}(\boldsymbol{X}_{kk} - \hat{\boldsymbol{v}}\,\bar{\boldsymbol{K}}) = p_B X_{BB} \tag{3}$$

where vector $\boldsymbol{X}_{kk} \geq 0$ represents their demands for fixed capital goods (i.e., $\boldsymbol{p}_{kk}\boldsymbol{X}_{kk}$ is their total gross investment and $\boldsymbol{p}_{kk}(\boldsymbol{X}_{kk} - \hat{\boldsymbol{v}}\,\bar{\boldsymbol{K}})$ is their total net investment), while X_{BB} is the quantity of securities issued for financing investments. At the end of the period, the owners of fixed capital goods have vector $(\boldsymbol{I} - \hat{\boldsymbol{v}})\bar{\boldsymbol{K}} + \boldsymbol{X}_{kk}$ of fixed capital goods while the total quantity of securities is $z\bar{B} + X_{BB}$.

The number of relationships (1)-(3) is $n_k + 2$, where n_k is the number of fixed capital goods.

[16] Diewert (1978, pp. 78-79) believes that this equality is incorrect, i.e., that the original Walras relationship $p_{kk}^i = \frac{p_k^i}{\frac{1}{p_B} + v^i}$ must be substituted by $p_{kk}^i = \frac{p_k^i(1 + \frac{1}{p_B})}{\frac{1}{p_B} + v^i}$. Diewert refers this relationship also to circulating capital goods (which would require $v^i = 1$ and $p_{kk}^i = p_k^i$) neglecting that Walras excludes them from the theory of capitalization and credit since they are the object of the theory of circulation and money. Walras' relationship is correct with reference to fixed capital goods the moment we consider that they are assumed to be used only from the period subsequent to that of their production, so that their present value is

$$p_{kk}^i = \frac{\pi_k^i}{1 + \frac{1}{p_B}} + \frac{\pi_k^i}{(1 + \frac{1}{p_B})^2} + \ldots = \frac{\pi_k^i}{\frac{1}{p_B}}$$

where net income π_k^i is $\pi_k^i = p_k^i - v^i p_{kk}^i$. We can extend the relationship under examination also to circulating capital goods by including the availability service (which takes into account that products are obtained after inputs are available): consequently we have $p_{kk}^i = \frac{p_k^i + p_k^{ia}}{\frac{1}{p_B} + v^i}$ where p_k^{ia} is the value of the availability service, $v^i = 1$, and $p_{kk}^i = p_k^i$, so that $p_k^i = \frac{p_k^i + p_k^{ia}}{\frac{1}{p_B} + 1}$ which requires $p_k^{ia} = \frac{p_k^i}{p_B}$ exactly the relationship proposed by Walras in the theory of circulation and money.

[17] Investment is not infinitely elastic if expected prices are not stationary. If the demand for fixed capital goods is not infinitely elastic and capital goods are not perfect substitutes, then equations (2) are substituted by demand functions of the type $\boldsymbol{D}_{kk} = \boldsymbol{F}_{kk}(\boldsymbol{p}_k, \ldots)$. Morishima (1977, pp. 100-22), on the contrary, maintains conditions (2) even when a non-infinitely elastic investment is introduced, consequently obtaining an inconsistent system.

2) Owners of circulating capital goods

The endowments of the owners of circulating capital goods are composed of assets represented by a vector $\bar{\boldsymbol{C}} = (\bar{C}^1,...,\bar{C}^{n_c})$ of quantities of goods produced in the preceding period and of liabilities represented by the quantity of money $\bar{M}$. During the period under examination, the owners of circulating capital goods offer their goods, while consumers and producers demand them using the whole quantity of money available to them. In fact, on the one hand, consumers and producers prefer not to save money since it would be better for them, respectively, to hold securities and not to borrow money. On the other hand, there is no advantage for the owners of circulating capital goods in holding a speculative store of goods (since they expect stationary prices). However, selling goods at current prices is indifferent to them: we assume that they sell the whole quantity of circulating capital goods during the period (i.e., the offer of these goods is determined and inelastic). This assumption can be easily justified taking into account some elements which have been excluded for the sake of simplicity (such as the cost of storage and the cost of issuing money).[18] Consequently, the owners of circulating capital goods offer, during the period, the whole quantity of goods and receive correspondingly the whole quantity of money, so that

(4) $$\bar{M} = \boldsymbol{p}_c\bar{\boldsymbol{C}}$$

where vector $\boldsymbol{p}_c = (p_c^{\,1},..., p_c^{\,n_c})$ represents the prices of circulating capital goods.

The owners of circulating capital goods invest (i.e., they demand circulating capital goods) by maximizing expected profit. Representing with $\boldsymbol{d}_c, s_M$ the quantities of demanded goods and of offered money of an investor, the expected profit is $\pi_c^e = \boldsymbol{p}_c^e\boldsymbol{d}_c - s_M$ under the finance constraint $s_M = \boldsymbol{p}_c\boldsymbol{d}_c$. Consequently,

$$\pi_c^e = (\boldsymbol{p}_c^e - \boldsymbol{p}_c)\boldsymbol{d}_c$$

The hypothesis of expectations of stationary prices implies that $\pi_c^e = 0$, that circulating capital goods are perfect substitutes, and that their demand is infinitely elastic at the current prices $\boldsymbol{p}_c$.[19] (Equation (4) and the condition that consumers and producers spend the whole quantity of money $\bar{M}$ also imply that the current total profit of the owners of circulating capital goods is zero, but it does not require that the profit of every owner is zero. We assume for simplicity that these possible profits, the sum of which is zero, do not affect consumers' incomes).

The last relevant equation concerning the owners of circulating capital goods is given by their total finance constraint

(5) $$\boldsymbol{p}_c\boldsymbol{X}_{cc} = \bar{M} + X_{MM}$$

where vector $\boldsymbol{X}_{cc} \geq 0$ represents their demands (i.e., $\boldsymbol{p}_c\boldsymbol{X}_{cc}$ is their total gross investment and $\boldsymbol{p}_c(\boldsymbol{X}_{cc} - \bar{\boldsymbol{C}})$ is the total net investment), and $\bar{M} + X_{MM}$ is the total amount of money issued at the end of the period (X_{MM} is the increment, positive or negative, in the quantity of money with respect to the preceding period). The number of relationships (4) and (5) are 2.

[18] When money is formed by bank deposits, as assumed in the following Section, the sale of all the inventories of circulating capital goods is convenient for their owners.

[19] Investment is not infinitely elastic if expected prices are not stationary. In this case the prices of capital goods produced in the period under examination and exchanged at the end of the period do not necessarily equal the prices of capital goods produced in the preceding period, available in stores and exchanged during the period.

3) Households

Households (indicated also as consumers) have endowments composed of personal capitals, of which they sell the services (labor), securities and money. They lend some of this to producers. The remaining part is used for purchasing consumer goods during the period under examination (these goods are sold by the owners of circulating capital goods). Thus, consumers demand consumer goods (during the period, with immediate payment in money); they lend money to producers (with payment of interests and reimbursement of the principal at the end of the period); they supply labor (to producers, with wages paid at the end of the period); they demand services of fixed capital goods (supplied by their owners with payment at the end of the period); and they demand securities and money in order to have an income in the future and the liquidity sufficient for buying consumer goods in the following period.

Let us introduce an intertemporal utility function for any consumer, of the type

$$U = \Phi(\boldsymbol{d}_k, \boldsymbol{d}_\ell, \boldsymbol{d}_c, \boldsymbol{d}_{k,1}, \boldsymbol{d}_{\ell,1}, \boldsymbol{d}_{c,1}, ..., \boldsymbol{d}_{k,t}, \boldsymbol{d}_{\ell,t}, \boldsymbol{d}_{c,t}, ...)$$

where vectors $\boldsymbol{d}_{k,t}, \boldsymbol{d}_{\ell,t}, \boldsymbol{d}_{c,t}$, with t = 1,... , indicate respectively the services of fixed capital goods, the labors and the circulating capital goods which the consumer plans to use in the t-th period after the one under examination. Taking into account that there are expectations of stationary prices, the liquidity constraints are: for the current period $\boldsymbol{p}_c\boldsymbol{d}_c = \overline{m} - s_M$, and for the following periods $\boldsymbol{p}_c\boldsymbol{d}_{c,t} = m_t - s_{M,t}$, with t = 1,… , where m_t is the quantity of money at the beginning of period t and $s_{M,t}$ is the amount lent to producers. The budget constraints (which concern payment at the end of each period) are: for the current time period

$$\boldsymbol{p}_k\boldsymbol{d}_k + \boldsymbol{p}_\ell\boldsymbol{d}_\ell + p_B d_B + \overline{m} + d_M = \boldsymbol{p}_\ell\overline{\boldsymbol{l}} + z\overline{b} + (1+i)s_M$$

and for the following periods

$$\boldsymbol{p}_k\boldsymbol{d}_{k,t} + \boldsymbol{p}_\ell\boldsymbol{d}_{\ell,t} + p_B d_{B,t} + m_t + d_{M,t} = \boldsymbol{p}_\ell\overline{\boldsymbol{l}}_t + b_t + (1+i)s_{M,t} \qquad t = 1,\ldots$$

where $d_{B,t}$ is the excess demand for securities at time t, $d_{M,t}$ is the excess demand for money, $\overline{\boldsymbol{l}}_t$ is the vector of available labors,[20] and b_t are the securities at the beginning of period t. There are also some constraints linking variables of successive periods: $m_t = m_{t-1} + d_{M,t-1}$ and $b_t = b_{t-1} + d_{B,t-1}$, with t = 1,… , $m_0 = \overline{m}$, $b_0 = z\overline{b}$, $d_{M,0} = d_M$, and $d_{B,0} = d_B$.

The maximization of the utility function with respect to $\boldsymbol{d}_k, \boldsymbol{d}_\ell, \boldsymbol{d}_c, d_B, s_M$, $d_M, \boldsymbol{d}_{k,1}, \boldsymbol{d}_{\ell,1}, \boldsymbol{d}_{c,1}, b_1, d_{B,1}, m_1, s_{M,1}, d_{M,1}, ..., \boldsymbol{d}_{k,t}, \boldsymbol{d}_{\ell,t}, \boldsymbol{d}_{c,t}, b_t, d_{B,t}, m_t, s_{M,t}, d_{M,t}, ...,$ subject to the aforementioned constraints leads to a choice[21] which can be expressed, for the current period (the only one relevant in the analysis of a temporary equilibrium), by the relationships

$$\boldsymbol{d}_k = \boldsymbol{f}_k(\boldsymbol{p}_k, \boldsymbol{p}_\ell, \boldsymbol{p}_c, i, z)$$

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[20] Vectors $\overline{\boldsymbol{l}}$ and $\overline{\boldsymbol{l}}_t$ are assumed to be data. However, we can assume a set of possible labors represented by a function $g(\boldsymbol{l}, ..., \boldsymbol{l}_t, ...) = 0$. Vector $\boldsymbol{l}$ will be determined by maximizing the utility function with respect to $\boldsymbol{l}$ (and all other variables) subject to $g(\boldsymbol{l}, ..., \boldsymbol{l}_t, ...) = 0$ (and all other constraints).

[21] The first order conditions are represented by the constraints and by the equations

$$\frac{\partial\Phi}{\partial\boldsymbol{d}_{k,t}} = \frac{1}{(1+i)^t}\lambda\boldsymbol{p}_k, \qquad \frac{\partial\Phi}{\partial\boldsymbol{d}_{\ell,t}} = \frac{1}{(1+i)^t}\lambda\boldsymbol{p}_\ell, \qquad \frac{\partial\Phi}{\partial\boldsymbol{d}_{c,t}} = \frac{1}{(1+i)^{t-1}}\lambda\boldsymbol{p}_c, \qquad \text{and} \qquad p_B = \frac{1}{i},$$

where λ is a Lagrange multiplier. Since $p_B = \frac{1}{i}$, money and securities are perfect substitutes.

$$\boldsymbol{d}_\ell = \boldsymbol{f}_\ell(\boldsymbol{p}_k, \boldsymbol{p}_\ell, \boldsymbol{p}_c, i, z)$$
$$\boldsymbol{d}_c = \boldsymbol{f}_c(\boldsymbol{p}_k, \boldsymbol{p}_\ell, \boldsymbol{p}_c, i, z)$$
$$p_B = \frac{1}{i}$$
$$s_M = \bar{m} - \boldsymbol{p}_c\boldsymbol{d}_c$$
$$\frac{1}{i}d_B + \bar{m} + d_M = z\bar{b} + (1+i)s_M - \boldsymbol{p}_k\boldsymbol{d}_k + \boldsymbol{p}_\ell(\bar{\boldsymbol{l}} - \boldsymbol{d}_\ell)$$

These relationships can be aggregated for all consumers. We obtain the demand functions for circulating capital goods

(6) $$\boldsymbol{X}_c^h = \boldsymbol{F}_c^h(\boldsymbol{p}_k, \boldsymbol{p}_\ell, \boldsymbol{p}_c, i, z)$$

the corresponding offer of money during the period under examination

(7) $$X_M^h = \boldsymbol{p}_c\boldsymbol{X}_c^h$$

the offer of loans to producers

(8) $$X_M^f = \bar{M} - \boldsymbol{p}_c\boldsymbol{X}_c^h$$

the demand functions for services of fixed capital goods

(9) $$\boldsymbol{X}_k^h = \boldsymbol{F}_k^h(\boldsymbol{p}_k, \boldsymbol{p}_\ell, \boldsymbol{p}_c, i, z)$$

the supply functions of labors

(10) $$\bar{\boldsymbol{L}} - \boldsymbol{X}_\ell^h = \bar{\boldsymbol{L}} - \boldsymbol{F}_\ell^h(\boldsymbol{p}_k, \boldsymbol{p}_\ell, \boldsymbol{p}_c, i, z)$$

the condition of equal returns for loans and securities

(11) $$p_B = \frac{1}{i}$$

and the demand for securities and money (available at the end of the period), which are perfect substitutes

(12) $$p_B X_{BB} + \bar{M} + X_{MM} = z\bar{B} + (1+i)X_M^f - \boldsymbol{p}_k\boldsymbol{X}_k^h + \boldsymbol{p}_\ell(\bar{\boldsymbol{L}} - \boldsymbol{X}_\ell^h)$$

The number of relationships (6)-(12) is $n_k + n_\ell + n_c + 4$.

4) Producers

Producers buy circulating capital goods, labors and services of fixed capital goods required as inputs and sell the circulating and fixed capital goods obtained as outputs. Since the circulating capital goods used as inputs are paid during the period (while outputs and other inputs are paid at the end of the period) producers must borrow money at the beginning of the period, which will be given back with the corresponding interests at the end of the period.

Producers maximize profit

$$\Pi_f = \boldsymbol{p}_c\boldsymbol{X}_{cc}^f + \boldsymbol{p}_{kk}\boldsymbol{X}_{kk}^f - \boldsymbol{p}_k\boldsymbol{X}_k^f - \boldsymbol{p}_\ell\boldsymbol{X}_\ell^f - (1+i)X_M^f$$

where vectors $\boldsymbol{X}_{cc}^f, \boldsymbol{X}_{kk}^f$ indicate the quantities of products; $\boldsymbol{X}_k^f, \boldsymbol{X}_\ell^f$ the demands for services of fixed capital goods and of labors, the amount of which equals in equilibrium the differences $\bar{\boldsymbol{K}} - \boldsymbol{X}_k^h$ and $\bar{\boldsymbol{L}} - \boldsymbol{X}_\ell^h$; and X_M^f the quantity of money borrowed, subject to the liquidity constraint

$$\boldsymbol{p}_c\boldsymbol{X}_c^f = X_M^f$$

where $\boldsymbol{X}_c^f$ indicates the demands for circulating capital goods, the amount of which equals in equilibrium $\bar{\boldsymbol{C}} - \boldsymbol{X}_c^h$, so that

(13) $$\boldsymbol{p}_c(\bar{\boldsymbol{C}} - \boldsymbol{X}_c^h) = X_M^f$$

and to the constraints which require all vectors to be nonnegative and compatible with the technical possibilities of production. Assuming for the sake of simplicity fixed coefficients of production, these constraints require

$$\begin{aligned} \boldsymbol{A}_{kc}\boldsymbol{X}_{cc} + \boldsymbol{A}_{kk}\boldsymbol{X}_{kk} &= \bar{\boldsymbol{K}} - \boldsymbol{X}_k^h \\ \boldsymbol{A}_{\ell c}\boldsymbol{X}_{cc} + \boldsymbol{A}_{\ell k}\boldsymbol{X}_{kk} &= \bar{\boldsymbol{L}} - \boldsymbol{X}_\ell^h \\ \boldsymbol{A}_{cc}\boldsymbol{X}_{cc} + \boldsymbol{A}_{ck}\boldsymbol{X}_{kk} &= \bar{\boldsymbol{C}} - \boldsymbol{X}_c^h \end{aligned} \tag{14}$$

where $\boldsymbol{A}_{kc}, \boldsymbol{A}_{kk}, \ldots$ are matrices of coefficients. Taking into account these constraints and the competitive condition by which maximal profit is zero, we obtain the following relationships

$$\begin{aligned} \boldsymbol{p}_k\boldsymbol{A}_{kc} + \boldsymbol{p}_\ell\boldsymbol{A}_{\ell c} + (1+i)\boldsymbol{p}_c\boldsymbol{A}_{cc} &= \boldsymbol{p}_c \\ \boldsymbol{p}_k\boldsymbol{A}_{kk} + \boldsymbol{p}_\ell\boldsymbol{A}_{\ell k} + (1+i)\boldsymbol{p}_c\boldsymbol{A}_{ck} &= \boldsymbol{p}_{kk} \end{aligned} \tag{15}$$

The number of relationships (13)-(15) is $2n_k + n_\ell + 2n_c + 1$.

System (1)-(15) is composed of $4n_k + 2n_\ell + 3n_c + 9$ equations,[22] two more equations than the variables, which are $\boldsymbol{X}_k^h, \boldsymbol{X}_{kk}, \boldsymbol{p}_k, \boldsymbol{p}_{kk}, \boldsymbol{X}_\ell^h, \boldsymbol{p}_\ell, \boldsymbol{X}_c^h, \boldsymbol{X}_{cc}, \boldsymbol{p}_c, X_M^h$, $X_M^f, X_{MM}, X_{BB}, i, p_B$ and *z*. Correspondingly, there are two dependent relationships since Walras' law holds both for the transactions during the period and for the transactions with payments at the end of the period. In fact, beside the usual dependence (among relationships (1), (2), (3), (5), (12), (13), (14) and (15)), we find that the sum of relationships (4), (8) and (13) is zero.[23]

III. Implications of the restated Walrasian theory of money

The theory presented in Section 3, even if based on Walras' principal hypotheses, differs substantially from the original one. It is the theory of circulation and money which differs, not that of capitalization and credit. In fact, Walras' original theory of capitalization and credit is obtained if we assume that households are also owners of fixed capital goods.[24] [25]

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[22] All relationships are indicated as equalities. Taking into account land, old fixed capital goods and that some goods could be free goods, some relationships would be represented as inequalities. This possibility is not introduced for simplicity: however, it is useful only when we must demonstrate the existence of a solution, a problem which goes beyond the scope of this work. With reference to land and old fixed capital goods, the number of which is here indicated with n_k', the n_k' corresponding variables in vector X_{kk} are equal to zero and the n_k' corresponding equations in subsystem (15) are strict inequalities, i.e., $\boldsymbol{p}_k\boldsymbol{A}_{kk} + \boldsymbol{p}_\ell\boldsymbol{A}_{\ell k} + (1+i)\boldsymbol{p}_c\boldsymbol{A}_{ck} > \boldsymbol{p}_{kk}$. If we like to take into account also the fixed capital goods of a new type (which are produced in the period under consideration for the first time), then their quantities in vectors $\bar{\boldsymbol{K}}$ and $\boldsymbol{X}_k^h$ are zero, as well as the corresponding coefficients $\boldsymbol{A}_{kc}$ and $\boldsymbol{A}_{kk}$, the corresponding equations in subsystem (2) do not determine the current prices of their services, but they indicate the expected prices for the future periods, and the corresponding functions $\boldsymbol{F}_k^h(.)$ in subsystem (9) are identically equal to zero: consequently, for the new fixed capital goods, there are no corresponding equations in subsystems (2) and (9) and no corresponding variables $\boldsymbol{p}_k$ and $\boldsymbol{X}_k^h$.

[23] Should there be only one good (circulating capital) and money, we find the following system

$$\begin{array}{lll} \bar{M} = p_c\bar{C} & p_c X_{cc} = \bar{M} + X_{MM} & X_c^h = F_c^h(p_c, i) \\ X_M^h = p_c X_c^h & X_M^f = \bar{M} - p_c X_c^h & \bar{M} + X_{MM} = (1+i)X_M^f \\ p_c(\bar{C} - X_c^h) = X_M^f & a_{cc}X_{cc} = \bar{C} - X_c^h & (1+i)p_c a_{cc} = p_c \end{array}$$

the solution of which requires

$$i = \frac{1}{a_{cc}} - 1 \qquad\qquad p_c = \frac{\bar{M}}{\bar{C}}$$

[24] If owners of fixed capital goods are identified with households, then relationships (1) and (3) must be merged with relationship (12). We obtain, also taking into account relationships (2) and (8)

The principal implications of the restated Walrasian theory of money are the following.

a) *There are two quantitative relations.* Relationships (4) and (5) are two monetary quantitative equations: the former concerns transactions during the period under examination (the circulating capital goods produced in the preceding period and stored are sold by their owners to consumers and producers); the latter concerns transactions at the end of the period (the circulating capital goods produced in the period are sold by producers to the owners of circulating capital goods). The causal nexus of these relations can be synthesized as follows. Relationship (4) indicates that the value of money (i.e. the general level of prices) depends on the quantity of money $\bar{M}$, which is a predetermined variable. Relationship (5) indicates that the quantity of money $\bar{M} + X_{MM}$ issued at the end of the period depends on the value of money (i.e. on the expected level of prices, which is predetermined and equal to the current one because of the assumption of stationary expected prices).[26]

b) *The theory of money is fully integrated with the theories of exchange, production and capitalization.*[27] The theory of money is integrated with the theory of exchange since the quantity of money, its value and the rate of interest influence consumers' choices, even if money is not directly useful. On the one hand, loans yield an income and, on the other hand, money is indirectly useful because of the liquidity constraints. The theory of money is integrated with the theory of production since producers are subject to the liquidity constraint and there are, among costs, the interests on monetary loans. The theory of money is integrated with the theory of capitalization since, at equilibrium prices, securities and money are perfect substitutes in consumers' portfolios and give the same yield. Also the analysis of stability of monetary equilibrium can be done as usual by considering the excess demand for stored circulating capital goods (produced in the preceding period), for labors and for services of fixed capital goods and the difference between selling prices and cost prices of products.[28]

$$\boldsymbol{p}_{kk}\boldsymbol{X}_{kk} + X_{MM} = \boldsymbol{p}_k(\bar{\boldsymbol{K}} - \boldsymbol{X}_k^h) + \boldsymbol{p}_\ell(\bar{\boldsymbol{L}} - \boldsymbol{X}_\ell^h) + i\bar{M} - (1+i)\boldsymbol{p}_c\boldsymbol{X}_c^h$$

which is the last of Walras' equations (2) (Walras, 1954, p. 279) once the variables considered by the theory of circulation and money are disregarded. The same goes for the other equations.

25 Walras' original theory identifies owners of fixed capital goods with households (so that saving is identical to investment). Other theories (like Keynes' theory) identify them with producers. Both these positions are compatible with the formulation presented in this paper and determine the same equilibrium conditions. The infinite elasticity of investment (which is not present in Keynes' theory) is not determined by a lacking distinction between the motivations to save and those to invest, but by the hypotheses of expectations of stationary prices and of perfect competition (with free entry and exit).

26 Naturally, complete causal nexus are not so simple as above. In the model with only one good and money introduced in footnote (23) we have

$$p_c = \frac{\bar{M}}{\bar{C}} \qquad\qquad \bar{M} + X_{MM} = p_c X_{cc}$$

where

$$X_{cc} = \frac{1}{a_{cc}}(\bar{C} - F_c^h(p_c, \frac{1}{a_{cc}} - 1))$$

27 Patinkin (1965, p. 571) asserts that Walras' theory of money is not fully integrated with the theory of production and capitalization and refers this missed integration to the lack of interdependence between the *tâtonnement* on the money market and that on the other markets.

28 The stability analysis (which is the modern version of the Walrasian *tâtonnement*) of the simple model introduced in footnote (23) takes into consideration the excess demand for the stored good

$$E_c = \frac{\bar{M}}{p_c} - \bar{C}$$

the excess demand for loans

$$E_M = p_c a_{cc} X_{cc} - (\bar{M} - p_c F_c^h(p_c, i))$$

and the difference between the selling price and the cost price of the product

c) *Securities and money are a veil.* Securities and money exist only because real goods are owned by households through other agents. If these agents (i.e., the owners of fixed capital goods and the owners of circulating capital goods) are not introduced and the property is given directly to households, securities and money do not exist. In this case for all the real goods we find equilibrium conditions which are identical to those obtained when there are also the owners of capital goods.[29] In this sense, securities and money are a veil. Anyhow, without friction, uncertainty and illusion, securities and money must perforce be a veil: they can neither improve nor worsen the allocation of resources, which is a Pareto optimum (the Pareto optimum of temporary allocations).

d) *Paper money or bank deposits?* In the formulation proposed in Section 3 money, which is paper money yielding nothing, is issued by the owners of circulating capital goods in order to finance the purchase of circulating capital goods. This assumption may seem unrealistic since it excludes the presence of banks. An alternative formulation consider the group of owners of circulating capital goods as representative of two types of agent: the *owners of stores* and the *banks*. They can be formally introduced and money defined as bank deposits, yielding an interest. This situation can be described in the following way. At the beginning of the period under consideration, banks' assets are represented by a credit towards the owners of stores, to whom they have lent the necessary sum for buying their initial inventories, and banks' liabilities are represented by a debit of the same amount towards households. This debt is represented by bank deposits. Assets and liabilities give a yield (in equilibrium at the current rate of interest), which is credited at the end of the period. During the period banks give credit to producers for their purchases of inputs. Whenever producers buy circulating capital goods from the owners of stores, the producers' debt increases and that of the owners of stores decreases to the same extent. When consumers buy consumer goods the debt of owners of stores decreases and consumers' deposits decrease to the same extent. At the end of the period, producers sell products and reimburse their debt, yield included. In the meantime, the owners of stores have also reimbursed their debt, but they, at the end of the period, buy products and in this way a new debt is created. At the end of the period, households receive the balance between proceeds and payments and the yield of securities and bank deposits, which composes their new deposits. The total amount of bank deposits will again be equal to the debt of the owners of stores. Equilibrium is not substantially modified by the distinction between owners of stores and banks. The only modification is represented by the course of the prices of circulating capital goods during the period: prices are not constant but increasing, according to the relationship

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$$G_c = p_c - (1+i)\,p_c a_{cc}$$

Assuming money to be the *numeraire,* we can refer to these variables respectively the adjustment of p_c, i and X_{cc}, in order to find conditions of stability.

If the owners of circulating capital goods are identified with households, so that money is excluded and loans are made in kind, then the model with only one good shows, assuming this good to be *the numeraire,* the functions

$$E_c = F_c^h(i) + a_{cc}X_{cc} - \overline{C} \qquad\qquad G_c = 1-(1+i)a_{cc}$$

therefore implying different conditions of stability.

[29] The equilibrium system without securities and money is composed of relationships (14), (15) and

$$\boldsymbol{p}_k = (\hat{\boldsymbol{v}} + i\boldsymbol{I})\,\boldsymbol{p}_{kk}$$

$$\boldsymbol{X}_c^h = \boldsymbol{F}_c^h(\boldsymbol{p}_k, \boldsymbol{p}_\ell, \boldsymbol{p}_c, i)$$

$$\boldsymbol{X}_k^h = \boldsymbol{F}_k^h(\boldsymbol{p}_k, \boldsymbol{p}_\ell, \boldsymbol{p}_c, i)$$

$$\overline{\boldsymbol{L}} - \boldsymbol{X}_\ell^h = \overline{\boldsymbol{L}} - \boldsymbol{F}_\ell^h(\boldsymbol{p}_k, \boldsymbol{p}_\ell, \boldsymbol{p}_c, i)$$

$$\boldsymbol{p}_{kk}\boldsymbol{X}_{kk} + \boldsymbol{p}_c\boldsymbol{X}_{cc} = \boldsymbol{p}_k(\overline{\boldsymbol{K}} - \boldsymbol{X}_k^h) + \boldsymbol{p}_\ell(\overline{\boldsymbol{L}} - \boldsymbol{X}_\ell^h) + (1+i)\,\boldsymbol{p}_c(\overline{\boldsymbol{C}} - \boldsymbol{X}_c^h)$$

This system is homogeneous with respect to $\boldsymbol{p}_{kk}, \boldsymbol{p}_c, \boldsymbol{p}_k, \boldsymbol{p}_\ell$.

$$p(\tau) = p\frac{1+i}{1+(1-\tau)i}$$

where i is the rate of interest and $0 \le \tau \le 1$ is the instant of the payment within the period under examination. This relationship is determined by an arbitrage condition taking into account that money gives a yield. With these prices nobody can profit by anticipating or postponing the sale of goods in the period. (For instance, if a sale is anticipated the seller cashes a lower price but he reduces in advance his debt, thus paying a lower interest).[30] [31]

e) *The length of the time period.* The length of the period is relevant in Walras' theory of money. It is a synthetic representation of the asynchronies which justify the existence of money. In fact, in this theory, if the length of the period is varied according to a factor θ, then, *ceteris paribus*, since agents need money for purchasing goods during the period, the quantity of money must vary according to the same factor; i.e., we need a quantity of money multiplied by θ in order to maintain unchanged prices.[32] The assumption that loans have a predetermined maturity and that their reimbursement coincides with the payments at the end of the period, even if unnecessary for giving value to money, influences its value, since otherwise agents would need a different quantity of money for operating current purchases. A more realistic description would require the specification of asyncronies, i.e. to specify the length and kind of productive process and the time intervals among all acts of production, consumption and investment.

If money is defined as bank deposits, instead of paper money, the predetermined maturity of loans is an unnecessary assumption and the theory can also be referred to continuous time. In this case, bank deposits are always equal to the value of circulating capital goods existing in storage or bought by producers (the quantities of these capital goods depend on the length of production processes and on asynchronies among production, consumption and investment).[33]

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[30] Owners of stores must sell all the inventories, otherwise, with expectations of stationary prices (i.e. with the same course of prices in the following period), they will incur losses.

[31] The simple model introduced in footnote (23) is modified in an irrelevant way. In fact, the first two equations are substituted by the following ones: for the owners of stores

$$p_c\overline{C} = \overline{D} \qquad\qquad p_c X_{cc} = \overline{D} + X_D$$

where $\overline{D}$ is their debt towards banks at the beginning of the period and $\overline{D} + X_D$ that at the end; for the banks

$$\overline{D} = \overline{M} \qquad\qquad \overline{M} + X_{MM} = \overline{D} + X_D$$

where $\overline{M}$ is again the quantity of money (now, bank deposits) at the beginning of the period and $\overline{M} + X_{MM}$ that at the end. In the remaining equations the quantity X_M^f (loans of households to producers) indicates the debt of producers towards banks before the sale of products (this debt corresponds to deposits of households). Paying attention to the sales of the stored good, by indicating with $X_{c,j}$ and τ_j respectively the quantity sold at instant τ_j and this instant (with $0 \le \tau_j \le 1$ and $\sum_j X_{c,j} = \overline{C}$), the proceeds of sellers are $p_c(\tau_j)X_{c,j}$ where $p_c(\tau_j)$ is the price at instant τ_j. With the course

$$p_c(\tau) = p_c\frac{1+i}{1+(1-\tau_j)i}$$

the owners of stores will have a null balance at the end of the period whatever instants τ_j and quantities $X_{c,j}$ may be. In fact, taking into account their initial debt, proceeds and interests, we find

$$\overline{D}(1+i) - p_c\sum_j X_{c,j}p_c(\tau_j)(1+(1-\tau_j)i) = (\overline{D} - p_c\overline{C})(1+i) = 0$$

[32] The above proportionality (which depends on the condition *ceteris paribus*) excludes seasonal production. Of course, if the length of the period has been increased by factor θ, then also the quantities of inputs, products and services change in the same proportion.

[33] In the case of the simple model introduced in footnote (23), assuming that the production lag is equal to a unitary period of time, the owners of stores have inventories

Conclusion

In the restated Walrasian theory of money, the quantity of money, its value and its movement depend on the assumed institutional framework, i.e. on the assumptions that the owners of circulating capital goods buy products at the end of each period and sell them, exclusively in exchange for money, during the following period; and that loans are reimbursed and all other payments are effective at the end of each period, etc. We have already emphasized the importance of these hypotheses which rule, even if they are not all equally relevant, the function of money in transactions. This theory explains how money can exist, have value and be relevant in agents' plans of utility and profit, without being a store of value beyond a period (i.e. money is a store of value only within a period), as is required in a world without friction, uncertainty and illusion. In fact, money is the exclusive purchasing power on circulating capital goods (the life of which lasts only one period); it is issued at the end of each period, when the owners of circulating capital goods buy them from their producers; and consumers and producers spend during each period the whole quantity of money existing at its start. Nevertheless, money is not socially useful, since it is a veil.

However, the function of money in transactions is insufficient for describing realistically actual monetary economies, not only because there is a permanent store of money, but also because monetary exchanges are not actually determined by an institutional constraint (stated in a theory by an assumption) but by mutual convenience of agents, which is not analyzed at all.[34]

$$C(t) = C(0) + \int_0^t (X_{cc}(\tau) - X_c^h(\tau) - X_c^f(\tau))d\tau$$

and debt towards banks

$$D^c(t) = D^c(0) + \int_0^t p_c(\tau)(X_{cc}(\tau) - X_c^h(\tau) - X_c^f(\tau))e^{i(\tau)(t-\tau)}d\tau$$

where $X_{cc}(\tau), X_c^h(\tau)$ and $X_c^f(\tau)$ are respectively the current intensity of production, of consumer demand and of producer demand and $i(\tau)$ is the current rate of interest for a unitary period of time; households have bank deposits (since $M(t) = D^c(t) + D^f(t)$ for every $t \geq 0$)

$$M(t) = M(0) + D^f(t) - D^f(0) + \int_0^t p_c(\tau)(X_{cc}(\tau) - X_c^h(\tau) - X_c^f(\tau))e^{i(\tau)(t-\tau)}d\tau$$

demand for consumption

$$X_c^h(t) = F_c^h(p_c(t), i(t))$$

and the budget constraint

$$\frac{dM(t)}{dt} + p_c(t)X_c^h(t) = i(t)M(t)$$

while for producers there are the relationships

$$a_{cc}X_{cc}(t+1) = X_c^f(t) \qquad\qquad p_c(t+1) = a_{cc}p_c(t)e^{i(t)}$$

$$D^f(t) = \int_{t-1}^t p_c(\tau)X_c^f(\tau)e^{i(\tau)(t-\tau)}d\tau$$

In case of a stationary equilibrium, where the only asynchrony consists of the production lag and money is necessary only in order to finance production, we have, for every t, $C = 0, D^c = 0$

$$X_{cc} = X_c^h + X_c^f \qquad X_c^h = F_c^h(p_c, i) \qquad a_{cc}X_{cc} = X_c^f$$

$$1 = a_{cc}e^i \qquad p_cX_c^h = iM \qquad M = D^f = \frac{e^i - 1}{i}p_cX_c^f$$

This system can be easily solved with respect to p_c, i, X_c^h, X_c^f, and X_{cc} (while M is a datum).

[34] This criticism applies to all models which assume a monetary constraint on exchanges (presented, together with others, by Arcelli, 1975). Schumpeter (1939, pp. 547-548) criticizes, with precise reference to the theory of money, this kind of assumption, which I would like to justify by considering that the elements, disregarded by the theory,

In other words, friction, uncertainty and illusion are not indispensable in describing a monetary economy. Walras' theory of money really describes a monetary economy without friction, uncertainty and illusion: an economy where money depends only on the transactions motive.[35] Walras' original version of the theory of money is inconsistent not because uncertainty is disregarded but because of the imperfections described in Section 2. The restatement proposed in Section 3 sets out to be a consistent version of Walras' theory. A theory of this type, moreover, can be enlarged by introducing, with friction and uncertainty, the demand for money due to precautionary and speculative purposes. The resulting theory would certainly be more complete than current monetary theories, which examine the patrimonial demand for money, disregarding circulation.

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which determine some features of the real word, can be assumed as institutional data. This method is very common in economics: for instance the same distinction between consumers and producers, currently assumed by theory, derives from ignoring the elements which actually determine it.

[35] Let us disregard the case where money is the only durable good, thus store of value, demanded for carrying purchasing power over the period.

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