AQUIFER ZONATION BASED ON JOINT INVERSIONS

Published in Geophysics:
ABSTRACT

Predictive groundwater modeling requires accurate information on aquifer characteristics. Geophysical imaging is a powerful tool for delineating aquifer properties at an appropriate scale and resolution, but it suffers from problems of ambiguity. One way to overcome such limitations is to adopt a simultaneous multitechnique inversion strategy. We have developed a methodology for aquifer characterization based on structural joint inversion of multiple geophysical data sets followed by clustering to form zones and subsequent inversion for zonal parameters. Joint inversions based on structural cross-gradient constraints require less restrictive assumptions than, say, applying predefined petrophysical relationships, and generally yield superior results. This approach has, for the first time, been applied to three geophysical data types in three dimensions. A classification scheme using maximum likelihood estimation is used to determine the parameters of a Gaussian mixture model that defines zonal geometries from the joint inversion tomograms. The resulting zones are used to estimate representative geophysical parameters of each zone, which are then used for field-scale petrophysical analysis. A synthetic study demonstrates how joint inversion of seismic and radar traveltimes and electrical resistance tomography (ERT) data greatly reduces misclassification of zones (down from 21.3% to 3.7%) and improves the accuracy of retrieved zonal parameters (from 1.8% to 0.3%) compared to individual inversions. We applied our scheme to a data set collected in northeastern Switzerland to delineate lithological subunits within a gravel aquifer. The inversion models resolve three principal sub-horizontal units along with some important 3-D heterogeneity. Petrophysical analysis of the zonal parameters indicates ~30% variation in porosity within the gravel aquifer and an increasing fraction of finer sediments with depth.

1.1 INTRODUCTION

Aquifer characterization is a prerequisite for predictive groundwater modeling. Traditionally, small-scale aquifer properties are derived from geological and hydrological investigations at a limited number of boreholes. These investigations may include simple lithological interpretations of drill chips/cuttings and laboratory determinations of physical properties (e.g., hydraulic conductivity) of retrieved cores. The sampling volume for such measurements is very limited, being restricted to the actual borehole positions. Furthermore, the assumption that laboratory-determined physical properties represent in-situ conditions may not be valid because of significant scaling effects, differences in confining pressure,
strain, and fracturing, and a natural sampling bias towards intact specimens. Geophysical well logging overcomes some of these problems and extends the sampling distance up to a maximum of approximately 1 m from the borehole. Nevertheless, upscaling information derived from laboratory measurements and well logs to help construct fluid-flow and transport models of heterogeneous formations does not always lead to accurate predictions [Scheibe and Chien, 2003].

Hydrologists often perform in situ pumping tests, which together with certain assumptions about aquifer homogeneity and isotropy, enable the aquifer to be characterized on a much broader scale. This constitutes the other extreme end member solution to the spatial scaling problem.

An intermediate scaling approach involves constraining hydrological models using appropriate crosshole and/or surface geophysical data. Geophysical imaging is particularly useful (and complementary) because it interrogates a much greater volume of the aquifer than simple drilling and logging, provides almost continuous subsurface sampling, and is capable of delineating heterogeneity. For example, Scheibe and Chien [2003] produce accurate predictions of bromide breakthrough curves using a combination of crosshole radar tomograms and flow-meter and slug-test data, whereas traditional hydrological data alone yield poor predictions. The success of this approach is attributed to the large spatial coverage and 0.1 – 1 m resolution of the radar tomograms combined with a strong correlation between radar wavespeed and effective porosity. For such approaches to produce meaningful results, there must be a strong empirical relationship between the tomographic (geophysical) parameter and the hydrological properties [Day-Lewis and Lane, 2004; Linde et al., 2006b]. Unfortunately, petrophysical relationships are often site-specific, such that they cannot be easily extrapolated to other locations or geologic conditions. Likewise, simplified theoretical relationships between geophysical parameters and hydrological properties have limited ranges of applicability and cannot always be directly applied to geophysical tomograms based on standard inversions [Day-Lewis and Lane, 2004].

Although tomograms obtained from a single geophysical data type may help to improve hydrological models, ambiguity often remains. The conventional way to reduce such nonuniqueness is to introduce external constraints, such as à priori geological information, smoothing, stochastic regularization, or inversion for a specific model type. An alternative approach for reducing model ambiguity is joint inversion of two or more different types of co-located geophysical data to produce a single integrated model. The power of this approach lies
in the complementary nature of the data and models that can be exploited to constrain the range of plausible hydrological interpretations. Since different geophysical methods (e.g., seismic and radar transmission and electrical resistance tomography [ERT]) are sensitive to different physical properties of an aquifer, joint inversion is traditionally achieved by imposing petrophysical relationships; multiple models are linked during the inversion by petrophysical equations with known parameter values [e.g., Tryggvason et al., 2002]. For example, since resistivity and dielectric permittivity both depend on porosity [e.g., Pride, 1994; Revil et al., 1998; Linde et al., 2006a], this style of joint inversion could directly yield porosity estimates. A drawback of such schemes is that general petrophysical models typically involve many parameters that are likely to vary spatially and are usually poorly known.

A less restrictive approach to joint inversion, presented independently by Haber and Oldenburg [1997] and Zhang and Morgan [1997], is to impose structural similarity between models during the inversion process without any explicit assumptions about petrophysical relationships, except that the geophysical parameters are assumed to vary at common locations. Gallardo and Meju [2003; 2004] developed this structural approach into a methodology for joint inversion by advocating that the gradients of a model could be used to quantify structures. They introduced the idea that the vector cross product of the gradients of two models should be forced to zero (implying similar directions of the gradient vectors) at each iterative step of a joint inversion. Changes in the two parameters at a given location must then be either zero, parallel, or anti-parallel. Gallardo and Meju [2003] applied this approach to field data and used the final models for visual zonation and interpretation. One key advantage of the structural approach to joint inversion is that scatter plots between different models provide less biased information about petrophysical parameters than those obtained using direct petrophysical approaches to joint inversion [Tryggvason and Linde, 2006]. Several researchers have further modified and applied the cross-gradients joint inversion approach [e.g., Gallardo and Meju, 2007; Linde et al., 2008], and it has recently been applied to more than two data types [Gallardo, 2007] and to 3-D data sets [Linde et al., 2006a; Tryggvason and Linde, 2006; Fregoso and Gallardo, 2009]. We extend here the methodology developed by Linde et al. [2006a] to three different types of 3-D data.

Rather than using inverted tomograms to delineate the detailed structure of an aquifer [e.g., Hubbard et al., 1999], it is often useful to integrate the information to define zones on a scale significantly larger than the inherent resolution of each tomogram. This allows us to
determine “effective” geophysical parameters that are only weakly affected by the original regularization constraints. The zones thus identified are assumed to have similar petrophysical characteristics [Hyndman and Gorelick, 1996; Hyndman and Harris, 1996; Eppstein and Dougherty, 1998] and hence are more useful in flow and transport modeling. Whereas applications of zonation and classification techniques are common in medical imaging and remote sensing, they are still in their infancy in geophysics [Avseth et al., 2001; Bedrosian et al., 2007]. In near-surface investigations, Tronicke et al. [2004] and Paasche et al. [2006] have applied K-means and fuzzy c-means clustering techniques to crosshole traveltime and attenuation tomograms. Neither method is well suited for zone identification and classification of strongly correlated parameters, which is a fundamental requirement when applied to structurally constrained joint inversions. By using maximum likelihood estimation to find the parameters of a Gaussian mixture model, our preferred clustering approach incorporates the covariance between the various parameters [Dempster et al., 1977; Mitchell, 1997]. As a consequence, this approach is explicitly aimed at determining zones for which there are strong correlations between the various parameters (e.g., Figure 4 in Gallardo and Meju [2003]).

In this contribution, we begin by explaining our inversion and clustering techniques and introducing the field test site in northern Switzerland. We then determine zonal models based on parameters estimated from structurally constrained joint inversions of comparable synthetic and field data sets, each comprising three different types of 3-D geophysical data (crosshole seismic, radar, and ERT). By fixing the zone boundaries defined by the joint inversion and clustering algorithms and re-inverting the same three data sets for uniform zonal parameter values (i.e., seismic and radar wavespeed and electrical resistivity), the effective parameters of each zone are established. As a final step, we use the estimated zonal parameters for the field example to infer hydrologically relevant properties, such as formation factor and relative variations in the distributions of fine materials (sils and clays).

1.2 METHODOLOGY

In this section we describe the main components of our scheme (i.e., joint inversion, cluster analysis and zonation, and zonal inversion). Figure 2.1 shows a flow-chart of the entire process from raw field data and à priori information to the final petrophysical properties of the aquifer.
Joint inversion

Our formulation and implementation of the inverse problem is illustrated in Figure 2.2 and follows closely the scheme outlined by Linde et al. [2006a; 2008]. Forward solvers are used to calculate seismic and radar traveltimes, electrical resistances, and the corresponding sensitivities. The traveltimes and raypaths are calculated in the high frequency limit using a finite-difference algorithm [Podvin and Lecomte, 1991; Tryggvason and Bergman, 2006], and the electrical responses and related sensitivities are computed using a finite-element solver implemented by Rückert et al. [2006].

The objective function $\Phi$ for $K$ different data sets is defined as

$$
\Phi = \sum_{k=1}^{K} w_k (\Phi_{d(k)} + 10^\varepsilon \Phi_{m(k)}) + \lambda \sum_{k=1}^{K} \sum_{l<k} T_{kl}
$$

(2.1)

with $\Phi_{d(k)}$ and $\Phi_{m(k)}$ being the data misfit and regularization term for data set $k$, and $T_{kl}$ is the sum of the absolute values of the cross-gradient function $t_{kl}$ between the models corresponding to data sets $k$ and $l$ (see below). The parameters $w_k$, $\varepsilon$ and $\lambda$ are the weighting factors for each data set, the regularization term, and the cross-gradients term, respectively.
1.2 Methodology

When individually inverting a single data set, only $\Phi_{d(k)}$ and $\Phi_{m(k)}$ contribute to the objective function (Equation 2.1 and Figure 2.2): $\Phi_{d(k)}$ is the misfit between the observed data and the data predicted by the wavespeed or resistivity-models and $\Phi_{m(k)}$ quantifies the model regularization that penalizes model structure in some sense. We use a stochastic regularization operator (having weight $10^\varepsilon$ relative to the data fit term) based on an exponential geostatistical model that penalizes model complexity and deviations from the initial input model [Linde et al., 2006a].

Joint inversion of two colocated data sets (e.g., radar and ERT) using a structural approach adds the third component to the objective function that enforces structural similarity [Haber and Oldenburg, 1997]. The structural similarity of two models $m_k$ and $m_l$ is quantified by calculating a normalized version of the cross-gradients function introduced by Gallardo and Meju [2003] at each location as [Linde et al., 2008]

$$t_{kl}(x,y,z) = \frac{\nabla m_k(x,y,z) \times \nabla m_l(x,y,z)}{m_{k,apriori}(x,y,z) \cdot m_{l,apriori}(x,y,z)}.$$  

(2.2)

The normalization of Equation 2.2 is with respect to the à priori models $m_{k,apriori}$ and $m_{l,apriori}$. Note that the denominator never goes to zero. By normalizing the cross-gradients function, the constraints for each model combination have, for a given relative change in the model properties, the same weight in the optimization. The linearized normalized cross-gradients function of $m_k$ and $m_l$ is added as a constraint at each iterative step by giving it a large weight in the linear system of equations [Linde et al., 2006a]. Paige and Saunders [1982] iterative conjugate gradient algorithm LSQR is used to minimize the objective function in a least-squares sense.

For three-method joint inversion, we calculate the cross-gradients function for every possible model combination to give three cross-gradients fields. These linearized cross-gradients constraints are imposed at each iterative step. Gallardo [2007] introduced an alternative approach that is based on the cross-product of a reference gradient (defined at each location as the strongest model gradient of all models in the previous iteration) and the gradients of each of the models being inverted for in the process.
Aquifer zonation based on joint inversions

**Figure 2.2.** Flowchart of the joint inversion scheme using cross-gradients constraints. $\Phi_d$ and $\Phi_m$ represent the sole contributions to the objective function $\Phi$ (Equation 2.1) in individual inversions. For joint inversion, coupling between the models is introduced by penalizing deviations of the cross-gradient function $t$ from zero during model optimization.

### 1.2.2 Zonation

We employ an unsupervised zonation algorithm to group model cells into zones distinguished by common physical characteristics, such that two or more zones with uniform properties are used to describe the distribution of geophysical parameters in a region of interest (e.g., an aquifer).

Defining the zones is a four-step process:

1. **Preprocessing:** The values of each input model (3-D tomogram) are scaled to a mean of 1 to avoid effects associated with the model units (e.g., km/s). The logarithm of electrical resistivity is used to compress the typically large range of values for this parameter. Classification needs reliable information from at least two methods for each cell and only model cells with ray coverage in either the seismic or radar forward models are used for the classification. This is guaranteed by sufficient ray coverage in one or both of the ray-based models plus the non-zero sensitivity of the ERT throughout the entire inversion domain. The decision of which cells to use for the cluster estimation could probably be improved by model appraisal (e.g., using the model covariance matrix) and might have to be adapted for surface-based data with generally coarser data coverage.

2. **Cluster estimation:** The clusters are modeled as a mixture of Gaussian functions with parameters that are automatically estimated on the basis of an expectation maximization
1.2 Methodology

After obtaining a geometrical model of the zones, we perform an overdetermined inversion to find the optimum parameter values (i.e., the seismic and radar wavespeeds and electrical resistivity) of each zone. The zonal inversion uses the same forward operators and preprocessed data as used for the joint inversion (shown by the dashed arrow in Figure 2.1), but instead of inverting for many thousands of model parameters it only inverts for one parameter value of each geophysical method in each zone. Starting from a homogeneous initial model, a linearized iterative inversion scheme is used to optimize the zonal parameters. There is no need for additional regularization, because the inverse problem is strongly overdetermined (e.g., 5000 data points to define 3 parameters). The parameter update is slightly reduced at each iterative step to stabilize the non-linear inverse process.

Convergence of this inversion process is both fast and robust. The final data misfit can be used as an indicator of how well the zonal model can explain the geophysical data. If only a few model parameters in the zonal model can describe the data almost as well as relatively
complex seismic, radar and ERT models with many thousands of parameters, then a zonal model is justified.

The inverted zonal parameters and any derived petrophysical properties are effective values on the scale of the zones, thereby providing a simple form of upscaling for use in future hydrological modeling. In most applications, it is highly unlikely that the underlying physical property distribution is made up of zones with uniform properties or overly smooth distributions. Constructing two end members of inversion models can help to understand the characteristics of the system under study.

1.3 FIELD SITE, PARAMETERS, AND PROCEDURES COMMON TO THE SYNTHETIC AND FIELD EXAMPLES

We have applied our methodology to a field site beside the Thur River in northern Switzerland, where the hydrological, ecological and biochemical effects of river restoration are currently being investigated [RECORD, 2011]. Our experiment is one component of a hydrogeophysical pilot study that targets a gravel aquifer in direct contact with an unrestored section of the river. We wish to estimate the spatial variability of geophysical and hydrogeological properties of the ancient river sediments forming the aquifer and to derive a 3-D zoned representation of the subsurface as a basis for future hydrogeophysical inverse modeling.

In this section we describe the following details that are common to the synthetic and field examples: geological/geophysical model, borehole geometry, recording configurations and processing parameters. The synthetic example helps to validate our new approach and guide the interpretation of the field data and associated models.

1.3.1 Experimental setup

Cores from boreholes across the field site reveal a laterally extensive three-layer structure with a 3-m-thick silty sand layer at the top, an intermediate-depth 7-m-thick gravel aquifer, and a thick impermeable clay aquitard at the base. The water table is normally at 4 m depth except during river flood events. For our experiment, the aquifer is accessed by four 11.4-cm-diameter fully-slotted PVC-cased boreholes located at the corners of a 5 × 5 m square, approximately 10 m from the river (Figure 2.3). Our analysis is concerned with this 5 × 5 m section of the 6-m-thick saturated part of the aquifer.
For the inversion of all three data sets, the area of interest is represented by a $7 \times 7 \times 6$ m volume containing a cubic mesh with 0.25 m edge lengths. The ERT model includes additional layers above and below this volume, but no regularization is applied across the boundaries to these layers. This is done because the boundaries are known to be sharp and because continuous regularization across these boundaries causes inversion artifacts within the gravel unit. A finer cubic mesh with 0.0625-m edge lengths is employed for the forward modeling of the seismic and radar traveltimes, whereas tetrahedra with 0.25-m edge lengths are used for the ERT forward modeling. Boundary effects in the ERT forward modeling were avoided by using a much larger domain than for the inversion and by using mixed type boundary conditions [Rücker et al., 2006].

Crosshole seismic, radar, and ERT data were acquired across all six planes between the four boreholes. Seismic data were recorded using source and receiver spacings of 0.25 m, whereas radar data were collected with source and receiver spacings of 0.5 m and 0.1 m, respectively. To ensure full symmetric radar coverage, the source and receiver antennas were interchanged and the experiment repeated for each plane. For the ERT survey, 9 electrodes at 0.7-m intervals were deployed in each borehole. We used two different types of electrode configuration [Bing and Greenhalgh, 2000]: the AB-MN configuration with two current electrodes (A, B) in one borehole and two potential electrodes (M, N) in a second borehole, and the AM-BN configuration with one current and one potential electrode in a common borehole and the other two electrodes in a second borehole. Data using all possible combinations of bipole size and position were acquired in each plane, resulting in a total of 2464 AB-MN and 7776 AM-BN measurements. To speed up the inversions, the 10240 measurements were reduced to the 5000 most information-rich values using an experimental design procedure (for details see Chapter 4).
Aquifer zonation based on joint inversions

Figure 2.3. Cross section of the field site located close to the Thur River in northeastern Switzerland (see inset). The gravel aquifer is intersected by four boreholes located at the corners of a 5 m x 5 m square approximately 10 m from the river. Our inversion domain has horizontal dimensions of 7 m x 7 m for all data and a vertical extent of 6 m for the seismic and radar data (solid red rectangle) and 12 m for the ERT data (dashed red rectangle).

1.3.2 Inversion parameters

During inversion, stochastic regularization was used with an exponential model [Deutsch and Journel, 1998]. To honor the subsurface layering evident in the borehole cores, without imposing excessive constraints, we used 1.5- and 0.75-m integral scales in the horizontal and vertical directions, respectively. Because no detailed geostatistical analysis had been carried out at this site, the integral scales were chosen in a pragmatic manner to be comparable to the resolving capabilities of the geophysical data but smaller than the borehole spacing. The integral scales were varied about the chosen values without significant changes in the final inversion results. The same geostatistical model was used for the indicator kriging.

Initial homogeneous input models were seismic wavespeed $\alpha = 2.05 / 2.05$ km/s, radar wavespeed $v_r = 76 / 76$ m/µs and electrical resistivity $\rho = 250 / 180$ Ωm for the synthetic / field examples. The standard deviation of the stochastic regularization was 10% of the input models for the seismic and radar experiments, corresponding to the expected variations in the field example and the variations used in the synthetic model. The ERT data were inverted for the logarithm of the resistivity, assuming a 20% standard deviation for the stochastic regularization.

Strong regularization (high $\epsilon$, see Equation 2.1) was employed for the initial inversion step and then progressively decreased with a specified step length after each iteration until the normalized root mean squared (RMS) misfit reached a predefined threshold (see Figure 2.2).
Successively decreasing $\varepsilon$ stabilizes the linearized inversion compared to using the final $\varepsilon$ throughout the inversion process. To compare the results between individual and joint inversion as well as between the different methods, the target normalized RMS misfit was set to a uniform 1.2. It was possible to invert for a normalized RMS misfit of 1.0, corresponding to the actual error level in the synthetic example, but this led to inversion artifacts that adversely affected the subsequent cluster classification.

All inversions reached the target misfit after 10-19 iterations. Convergence could be achieved with fewer iterations by decreasing $\varepsilon$ at a higher rate, but this resulted in higher values of the cross-gradients function in the final model [see Linde et al., 2008]. The value of the $\varepsilon$ was the same for all data sets during joint inversion, but each data set was weighted differently in the objective function and when calculating the model updates. The data set weights $w_I - w_J$ were applied to the data misfit and the regularization of the corresponding model (Equation 2.1). They were initialized to compensate for the different number of measurements in each data set (in inverse proportion) and varied until the 3-D tomograms of all methods predicted the data equally well. Figure 2.4 illustrates the process we used to determine $w_I - w_J$ and Table 2.1 shows the final inversion parameters for the synthetic and field examples. Tests using all possible combinations of two-method joint inversions and varying the inversion parameters gave very similar results to what is presented below for the three-method joint inversion, thus demonstrating the robustness of the methodology and each sequential part of it.

![Flow chart of the procedure to determine the relative weights $w_I - w_K$ of the $K$ different data sets for joint inversion. The threshold of the normalized RMS was set to 1.2 and the tolerance between the methods was chosen as $\text{tol} = 0.04$.](image-url)
Tables 2.1. Inversion parameters for the synthetic and field examples. In the column and row headers: S - seismic; R - radar; E - ERT individual inversions; SRE - joint inversion of all three methods.

<table>
<thead>
<tr>
<th>Data set</th>
<th>geophys. method</th>
<th>weight $w_k$</th>
<th>$\varepsilon$</th>
<th>iterations</th>
<th>RMS misfit mean</th>
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<td></td>
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<td>$R$</td>
<td>$E$</td>
<td></td>
<td></td>
</tr>
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<td>$S$</td>
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<td>1.10</td>
<td>10</td>
<td>1.19</td>
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<tr>
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<tr>
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<td>1.08</td>
<td>10</td>
<td>1.20</td>
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<tr>
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<td>16</td>
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<tr>
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<td>10</td>
<td>1.20</td>
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<tr>
<td>field</td>
<td>$R$</td>
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<td>1.20</td>
<td>10</td>
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<tr>
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<td>0.88</td>
</tr>
</tbody>
</table>

1.4 SYNTHETIC EXAMPLE

The synthetic input model (Figure 2.5) was chosen to mimic the field situation. It consists of three sub-horizontal layers, of which the middle layer has the highest seismic and radar wavespeeds and resistivity. Its thickness varies from 2.5 m in one corner to 1 m in the opposite corner. The seismic wavespeed $\alpha$ and resistivity $\rho$ are lowest in the bottom layer, whereas the radar wavespeed $v_r$ is lowest in the top layer. For the ERT modeling, an additional top layer with a resistivity of $\rho = 1000 \Omega\text{m}$ representing the unsaturated zone, and a bottom layer with a resistivity of $\rho = 25 \Omega\text{m}$ representing the clay aquitard, are added.

Source, receiver, and electrode positions, as well as the measuring configurations were identical to the field example. The synthetic data were created with the same forward solvers and grid as used for the inversions. The 2661 seismic and 5584 radar traveltimes were contaminated with 1% and the 5000 apparent resistivities with 3% uncorrelated Gaussian noise prior to inversion.

1.4.1 Individual and joint inversion results

The three data sets are inverted separately and jointly. All resulting 3-D tomograms (Figure 2.6) recover the main features of the synthetic input models (Figure 2.5), albeit with somewhat gradual rather than abrupt transitions between the layers. The seismic and radar models obtained by individual inversion (Figure 2.6a and b) clearly resolve the high wavespeed center region. Low wavespeed parts are less well resolved, especially for the radar model. This is mainly due to the limited ray coverage, particularly crossing rays, in the top
and bottom regions of the model. The individual ERT inversion (Figure 2.6c) resolves the two low resistivity zones, but the high resistivity layer in the middle is not continuous, as it should be. This poor performance in the center is attributed to current channeling in the high conductivity zones above and below and to the much higher sensitivities around the electrodes.

The 3-D seismic and radar tomograms resulting from joint inversion (Figure 2.6d and e) are very similar to the individual inversion tomograms, but the values of the low wavespeed regions are closer to the true values. The improvement in the joint inversion ERT tomogram (Figure 2.6f) is more pronounced, because the radar and seismic data help to constrain the geometry of the more resistive middle layer. Despite the restriction of rays to the acquisition planes and the concentration of ERT sensitivities around the electrodes, the homogeneous layers are well retrieved in 3-D, including the regions between the acquisition planes. The values of the cross-gradients function between the models were decreased by more than a factor of 100 compared to the individual inversions.

An informative view of the difference between models obtained from the individual and joint inversions is supplied by the scatter plots of Figure 2.7a-c and e-g. Whereas the scatter plots derived from the individual inversion show no evidence of clustering (Figure 2.7a-c), those from the joint inversion are sharply defined and relatively easy to classify (Figure 2.7e-g).

Figure 2.5. Input model used to create the synthetic data. It consists of three layers, the middle one of which has high seismic and radar waves speeds \( \alpha \) and \( v_r \), and a high resistivity \( \rho \) relative to the others. The thickness of this middle layer varies between 1 and 2.5 m at opposite corners. The synthetic data were contaminated with Gaussian noise (1% for seismic and radar traveltimes, 3% for apparent resistivities) before inversion. The boreholes used for the measurements are located at the four corners of the input model.
Figure 2.6. Results of (a-c) individual and (d-f) joint inversions of the synthetic data sets. All models fit the data with a normalized RMS misfit of 1.2 (1.0 corresponds to the error level). The main layering of the input model (see Figure 3) is observed in all models. The two seismic (a and d) and two radar (b and e) models are very similar, but a clear improvement is observed in the ERT model obtained by joint inversion (f) compared to the individual inversion model (c).

1.4.2 Classification

The scatter plots are now used for cluster estimation and zonation. The final number of zones was set to the true value of three for the synthetic study. Differences in the scatter plots are reflected in the classifications. The 50% confidence ellipsoids for each cluster are much smaller and more needle-like for the joint inversion (Figure 2.7h) than for the individual inversion results (Figure 2.7d). Outputs of the classification algorithm are the zonal models shown in Figure 2.8. The zonation based on the joint inversion tomograms (Figure 2.8c) is a much better reconstruction of the input model (Figure 2.8a) than that based on the individual inversion models (Figure 2.8b). Misclassification is only 3.7% for Figure 2.8c compared with 21.3% for Figure 2.8b. It is remarkable that the zones defined from the joint inversion tomograms are geometrically continuous, even though the positions of the cells are not considered during classification.
1.4 Synthetic example

Figure 2.7. Scatter plots for the models obtained by (a-c) individual and (e-g) joint inversion together with visualizations of the automatically determined clusters used for zonation of the models obtained by (d) individual and (h) joint inversion. The larger triangles and circles in (a-c) and (e-g) show the true parameter values of each zone in the input model. The colors of the symbols in the scatter plots and cluster representation correspond to the respective zonal model in Figure 2.8.

1.4.3 Zonal inversion

The inverted zonal parameters and RMS misfits are shown at the base of Figure 2.8. The parameter values for each zone are much better retrieved through zonal inversion than by averaging the models in Figure 2.6 for each zone. The zonal model from joint inversion predicts the data with similar RMS misfits as for the 3-D tomograms (1.1-1.5), but the zonal model from individual inversions fails to do so (RMS misfits of 1.6-3.0).

The inverted zonal parameters reproduce the true values with a deviation of only 0.3% when using the zones derived from the joint inversion tomograms, whereas the deviation is 1.8% when using the zones derived from the individual inversion tomograms. A deviation of almost 2% in the parameter estimation is quite significant when compared to the 10-20% variation in the parameters of the synthetic input models (Figure 2.5).

The misclassification rates and matches of the geometry and parameter values (see images and tables in Figure 2.8) for the synthetic study demonstrate the superior performance of the joint inversion scheme vis-à-vis the individual inversion approach. For field data with unknown zone geometries and parameters, the zonation has to be judged on the basis of the RMS data misfit and by visual inspection. The performance of the method applied to field data is investigated in the next section.
Figure 2.8. The (a) true zonal model (see Figure 5) and those derived by (b) individually and (c) jointly inverting the synthetic data. The zonal models derived from the individual and joint inversions have misclassification rates of 21.3% and 3.7%, respectively.

1.5 FIELD EXAMPLE

1.5.1 Measurements

Our field data were acquired with state-of-the-art equipment. A sparker source was used to generate seismic waves with a center frequency of about 1 kHz, and hydrophones and a GEODE system were used to record the seismic data at a sampling rate of 21 µs. Very strong signals caused the seismic waveforms to be clipped, but the first arrivals could clearly be identified (e.g., Figure 2.9a). Crosshole radar data at a 0.4 ns sampling rate were acquired using a RAMAC 250 MHz system, which at our site had a center frequency of ~100 MHz with energy in the 50-170 MHz frequency range (e.g., Figure 2.9b). ERT resistances were recorded using a Syscal Pro resistivity meter. Borehole deviations were measured with a deviation probe using a three-axis fluxgate magnetometer for bearing and a three-axis accelerometer for inclination. Corrections for the borehole deviations were critical for the traveltime inversions.

Preprocessing of the seismic and radar data included manual traveltime picking and assignment of the correct source-receiver positions in the deviated boreholes. A total of 2661 seismic and 5584 radar traveltimes could be reliably picked. Seismic traveltimes ranged between 2.2 and 4.3 ms and radar traveltimes ranged between 60 and 116 ns, with estimated picking errors for both data sets of ~1%.
The ERT data were strongly influenced by the resistivity contrast between the borehole fluid and the formation, such that correction factors derived from modeling with and without the boreholes had to be applied (for details see Chapter 4). A 2.5% error in the apparent resistivity data was assumed in the subsequent inversions. A frequency polygon plot of apparent resistivities before and after application of the borehole-fluid corrections is displayed in Figure 2.9c.

![Figure 2.9](image)

**Figure 2.9.** Typical raw (a) seismic and (b) radar source gathers for a source depth of 6.75 m. Red dots in (a) and (b) represent calculated forward responses of the final models obtained by joint inversion (see Figure 2.10d and e). (a) Although the seismic data were clipped, first arrivals could be reliably picked. (b) Picked first arrivals in the radar data do not include refracted waves through the unsaturated high wavespeed layer above 4 m; for the displayed source gather this means neglecting data collected above 5 m depth. (c) Frequency-polygon (histogram) of apparent resistivities plotted for raw and borehole-effect-corrected data.

### 1.5.2 Individual and joint inversion results

The models for the three parameter types obtained from the individual inversions contain very similar features, the most prominent being the high wavespeed and high resistivity layer in the middle of the domain (Figures 2.10a-c and 2.11a-c). Correlations between the models are relatively high (correlation coefficients > 0.6), even though each inversion is fully independent. This strong correlation indicates that all three methods sense the same geological/hydrological units, thus justifying the application of joint inversion to these data sets.
Seismic and radar tomograms from the three-method joint inversion (Figures 2.10d-e and 2.11de) are very similar to the individually inverted ones (Figures 2.10a-b and 2.11a-b), whereas resistivities in the jointly inverted ERT tomogram (Figures 2.10f and 2.11f) are noticeably more continuous than in the individually inverted one (Figures 2.10c and 2.11c). In similar fashion to the synthetic case, the magnitude of the cross-gradients between the models were reduced by more than a factor of 100 compared to the individual inversions. Although the images in Figure 2.10 are layered, when viewed from different directions (e.g., Figure 2.11) some pronounced 3-D heterogeneity is apparent. This heterogeneity is evident in all individual inversion tomograms (see Figure 2.11a-c), even though the sensitivity patterns of the traveltime and ERT data are fundamentally different with respect to location relative to the borehole.

The field example scatter plots in Figure 2.12 reveal the same variations between individual and joint inversion results as seen in the synthetic example (Figure 2.7). Although the scatter plots in Figure 2.12a-c show a general positive correlation between the geophysical parameters, the evidence for clusters is weak. By contrast, the joint inversion scatter plots in Figure 2.12e-g reveal distinct linear features.
Figure 2.11. As for Figure 10, but viewed from a different direction. There are clear 3D structures and the individual inversions (a-c) show that all main features are detected independently by the three methods.

Figure 2.12. Scatter plots for the models obtained by (a-c) individual and (e-g) joint inversion together with visualizations of the automatically determined clusters used for zonation of the models obtained by (d) individual and (h) joint inversion. The scatter plots obtained from the individual inversion models are rather diffuse. They demonstrate a generally positive correlation between the geophysical parameters. In contrast, the scatter plots from the joint inversion models show well-defined linear correlations. (d) and (h) The different character of the scatter plots is also observed in the cluster visualizations. The large yellow cluster in (h) “collects” all of the poorly defined scatter points. These cells are reclassified by geostatistical interpolation (indicator kriging).
1.5.3 Classification and zonal inversion

Application of the classification algorithm to the scatter plots in Figure 2.12a-c and subsequent zonal inversion yields a high wavespeed and high resistivity zone in the center of the aquifer, but it cannot distinguish the regions at the top and bottom of this zone from each other (Figure 2.13a and d). Increasing the number of clusters does not improve this result.

To achieve the most meaningful classification of the scatter plots in Figure 2.12e-g, the results for different numbers of clusters were compared. We obtained the best result using four clusters, three of which are meaningful. The fourth is a “collector” cluster (yellow region in Figure 2.13b and e) in which most poorly resolved cells are placed. These poorly resolved cells are mostly found along the top or bottom of the inversion domain. Rather than leave gaps in these regions, we interpolate/extrapolate values from adjacent areas using the indicator kriging method. The resulting three zones in Figure 2.13c and f (referred to as model 1) are overall spatially aligned and this model is used in the subsequent interpretation.

The RMS misfits that result from the zonal inversion (see tables at the base of Figure 2.13) are very similar for all zoned models; the relatively low 1.4-1.5 misfit for the seismic and radar wavespeeds and 2.8 - 3.5 for the ERT resistivities is largely controlled by the common definition of the high wavespeed/resistivity zone in the center. Separating the top and the bottom layer does not significantly improve the misfit, even though the \( v_r \) and \( \rho \) values vary between these layers. The low RMS misfit for the ray-based methods indicates that a zoned wavespeed model is reasonable, whereas the higher RMS misfit for the ERT values indicates that smaller scale resistivity variations are necessary to fit the data adequately.

1.6 HYDROGEOPHYSICAL INTERPRETATION

1.6.1 Petrophysical analysis

The radar and resistivity values of model 1 (Figure 2.13c and f) for each zone (see table at the base of Figure 2.13f) are used as input to our petrophysical analysis along with the following parameters: (i) the resistivity of the pore water \( \rho_w = 27 \, \Omega m \) (established from measurements in a nearby borehole at the time of the survey), (ii) the relative permittivity of the matrix \( \kappa_s \), which we cannot determine exactly but is unlikely to have strong variations within the gravel aquifer (see later), and (iii) the cementation factor \( m \) which we take to be 1.5-1.6 [Lesmes and Friedman, 2005].
1.6 Hydrogeophysical interpretation

Figure 2.13. Zonal models based on the (a and d) individual and (b and e) joint inversions shown in Figures 2.10 and 2.11 viewed from two different directions. The final zonation based on the models obtained by joint inversion (model 1) in (c) and (f) is described in the text.

Seismic wavespeed is not explicitly included in the analysis. In unconsolidated environments, both seismic and radar wavespeeds are a function of porosity, but porosity estimates from seismic wavespeed are not as well constrained as those from radar wavespeed. Comparison of the seismic and radar wavespeeds with the Hashin-Shtrikman bounds [Hashin and Shtrikman, 1963] indicates a very well connected pore space [Linde and Doetsch, 2010] and thus motivates our choice of a relatively low cementation factor.

We use the petrophysical model of Pride [1994] to relate the relative permittivity

\[ \kappa = \frac{c^2}{v_r^2} \quad (c = 300 \text{ m/µs}) \]  

(2.4)

to the formation factor \( F \) and porosity \( \phi \), linked by

\[ F = \phi^{-m}. \]  

(2.5)

in the following way:

\[ \kappa = \frac{1}{F}[\kappa_v + (F - 1)\kappa_r]. \]  

(2.6)
Aquifer zonation based on joint inversions

Here $\kappa_w = 84$ is the relative permittivity of water at 10°C [Eisenberg and Kauzmann, 1969] and $\kappa_s$ is the relative permittivity of the solid matrix. In the presence of fine-grain sediments (e.g., clays and silts), the electrical resistivity $\rho$ can be related to $F$ through a modified form of Archie's law [Linde et al., 2006a]

$$\frac{1}{\rho} = \frac{1}{F} \left[ \frac{1}{\rho_w} + (F-1)\sigma_s \right],$$

where $\sigma_s$ is surface conductivity. Surface conductivity occurs as a result of the electrical triple layer that forms at the interface between grains that comprise the sediment matrix and water. It is most prominent in materials with large specific surface areas such as clays and silts [e.g., Revil et al., 1998].

We solve Equation 2.6 for $F$ (and $\phi$ using Equation 2.5) and then use Equation 2.7 to estimate $\sigma_s$. To estimate $F$ and $\phi$ we have to assume a value for $\kappa_s$. We use $\kappa_s = 8.0$ to force surface conductivity $\sigma_s > 0$ in all three zones. The value of $\kappa_s$ is not established at this site, although it is known to lie within a restricted range for the lithologies under consideration. It is also reasonable to assume that variations of $\kappa_s$ within the gravel aquifer are likely to be quite small. Whatever value is used should not affect the relative variations in the deduced petrophysical parameters, but absolute values should be interpreted with caution. The range for $\phi$ in Table 2.2 corresponds to the likely variation in cementation factor $m$.

1.6.2 Interpretation

Two main findings result from this analysis (Table 2.2): (i) there is a distinct increase in surface conductivity $\sigma_s$ with depth and (ii) porosity $\phi$ is significantly lower in the middle layer. The bottom layer was found to have a total average conductivity of 6 mS/m (167 $\Omega$m), of which surface conductivity $\sigma_s$ is predicted to contribute about one third (2.1 mS/m). The increase in $\sigma_s$ can be attributed to a higher clay/silt fraction, demonstrating that it cannot be neglected in this sedimentary setting. Application of the conventional Archie’s law [Archie, 1942] to the resistivity values for the bottom layer would overestimate $\phi$ by 25%.

A ~30% variation of $\phi$ between the middle and the top and bottom layers is quite well resolved. Small scale variations are expected to be even larger. In contrast, the differences between $\phi$ in the top and bottom layers are not very well defined. For example, the differences in radar wavespeed can be explained by variations in the cementation factor (see Table 2.2). This possibility is supported by the very small differences in seismic wavespeed between the
upper and lower zones, which imply similar porosities assuming that there are no major differences in lithology \citep[e.g.,][]{Carcione2007}.

Although porosity is not well defined by our data and analysis, the more relevant parameters as far as fluid transport is concerned are the well-constrained formation factor $F$ and surface conductivity $\sigma_s$ \citep[e.g.,][]{Revil1999}. The hydrological implications of our results are being investigated with ongoing time-lapse ERT measurements.

**Table 2.2.** Result of the petrophysical analysis.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$F$</th>
<th>$\sigma_s$ [mS/m]</th>
<th>$\phi$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top (green)</td>
<td>7</td>
<td>0.2</td>
<td>26 - 29</td>
</tr>
<tr>
<td>Center (red)</td>
<td>12</td>
<td>0.6</td>
<td>19 - 21</td>
</tr>
<tr>
<td>Bottom (blue)</td>
<td>9</td>
<td>2.1</td>
<td>23 - 26</td>
</tr>
</tbody>
</table>

**1.7 DISCUSSION**

The tomographic models from the joint inversion and the corresponding zonal models are complementary representations of the situation at our field site. Models obtained from joint inversion display small-scale variability, but these variations are strongly affected by the regularization applied. As a consequence, they cannot be used directly to determine quantitative petrophysical values from theoretical models such as those described by Equations 2.6 and 2.7 \citep{Day-Lewis2004}. By comparison, the zonal inversion based on structures determined from clustering the joint inversion results yields physical parameters (e.g., seismic wavespeed) for relatively large zones. Such effective parameters are suitable for petrophysical analysis at field scales, but because small-scale variability is neglected the results are only meaningful if large-scale zones dominate the physical property fields.

The data misfit of the zonal parameter estimates is a measure of confidence in the zonal representation. For our field example, this implies that there are only minor seismic and radar wavespeed variations within the zones, because the zonal models can fit the data relatively well with a normalized RMS misfit of 1.4. In contrast, additional small-scale variability or vertical trends are probably needed to improve the fit to the ERT data (RMS misfit is a comparatively poor 3.5 for the zonal inversion values). This high misfit can also partly be caused by small errors in the assumed location of the water table and the clay aquitard, because no electrodes were located outside the saturated gravel.
1.8 CONCLUSIONS

We have presented a methodology for hydrogeophysical aquifer characterization based on cross-gradients joint inversion of 3-D crosshole seismic, radar, and ERT data followed by classification of zones and an over-determined inversion for zonal parameters. A zonation approach based on Gaussian mixtures was used to identify zones in the inversion models. Our synthetic example demonstrates how joint inversion reduces the misclassification rate from 21.3% for the individual inversions to 3.7% for the three-method joint inversion. The joint inversion zonal models also provide much better estimates of the zonal parameters (0.3% error compared to 1.8% using the individual inversion tomograms).

Our strategy of jointly inverting three types of 3-D data was applied to an active gravel aquifer adjacent to the Thur River in northern Switzerland. Clustering of the joint inversion tomograms produced noticeably better results than clustering of the individual inversion tomograms, primarily because of the clearer scatter plots obtained from the joint inversion models; the Gaussian mixture cluster estimation technique capitalized on this decreased scattering. The joint inversion and zonation models are complementary. The smooth joint inversion tomograms include information about lateral variability and general trends (e.g., decreasing resistivity with depth). The zonal representation summarizes important geometrical information about the aquifer and it enables petrophysical analysis at the field scale. The validity of the zonation can be assessed by zonal inversion for each method.

At our field site, we found three different sub-units within the gravel aquifer. The relative variation in porosity was estimated to be ~30% and the percentage of fine materials was found to increase with depth. The geometries and properties of the aquifer subunits determined here will be the starting point for hydrogeophysical modeling that will include data from extensive ongoing time-lapse ERT experiments at the same field site.
REFERENCES


1.8 Conclusions


References


Conclusions


References


tomography (Zugspitze, German/Austrian Alps), Journal of Geophysical Research-Earth Surface, 115, F02003.


1.8 Conclusions


1.8 Conclusions


References


1.8 Conclusions


